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EXISTENCE AND STABILITY OF SOLUTION IN BANACH SPACE FOR AN IMPULSIVE SYSTEM INVOLVING ATANGANA–BALEANU AND CAPUTO–FABRIZIO DERIVATIVES

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Abstract

This paper investigates the necessary conditions relating to the existence and uniqueness of solution to impulsive system fractional differential equation with a nonlinear p-Laplacian operator. Our problem is based on two kinds of fractional order derivatives. That is, Atangana–Baleanu–Caputo (ABC) fractional derivative and the Caputo–Fabrizio derivative. To achieve our main aims, we will first convert the proposed impulse system into an integral equation form. Next, we prove the existence and uniqueness of solutions with the help of Leray–Schauder's theory and the Banach contraction principle. We analyze the operator for continuity, boundedness, and equicontinuity. Further, we investigate the stability solution to the proposed impulsive system by using stability techniques. In the last part, we demonstrate the results via an illustrative example for the application of the results.

Keywords: Atangana–Baleanu–Caputo Derivative; Caputo–Fabrizio Derivative; Hyers–Ulam Stability; Existence and Uniqueness.

1. INTRODUCTION

In recent years, mathematicians and modelers have reacted enthusiastically to fractional calculus. It has recently proven to be a powerful tool for modeling a wide range of physical phenomena and an effective tool for describing memory and hereditary qualities in various processes. Fractional calculus has become an essential topic in multiple fields like electrochemistry, anomalous diffusion process, bioengineering, and fluid flow. For more details, the readers can see Refs. 1–6. Some scientists have expanded fractional calculus using Riemann–Liouville and Caputo derivatives and other derivatives. We should note that these two derivatives have a kernel with singularity. Also, they can be viewed on the one hand as the convolution of a derivative of a given function with the power function for Caputo case. For Riemann–Liouville case, it can be viewed as the derivative of a convolution of a given function with a power function.[7] Caputo and Fabrizio[8] presented a fractional-order derivative based on the exponential functions to process the singular kernel problems; nevertheless, their derivatives don't have a singular kernel. They established that their derivatives were appropriate for some types of physical problems. Atangana and Baleanu[9,10] presented two generalized fractional derivatives that relied on the Mittag-Leffler function to solve the problem of non-singular and non-local kernels. This definition upheld Caputo–Fabrizio's, which is on an exponential function. For more details, the reader can see Refs. 11–20. Newly, the study of impulsive FDEs has attracted a lot of interest from researchers on different types of FDEs. Researchers have presented many studies about the EUS of a solution to impulsive FDEs. For example, see Refs. 21–32. To our knowledge, no one has researched for existence and uniqueness of impulsive FDEs involving Caputo–Fabrizio's derivative and ABC-derivative. The motivation behind our study for an impulsive system of differential equations with non-integer order is that most of the models in applied fields nowadays involve fractional order derivatives and fractional order integrals. Due to the challenging process of determining the exact solution, we analyze stability solutions in our study. As a result, stability analysis is essential and useful in various applications, like optimization, numerical analysis, etc. Other significant types of stability analysis are UH–Rassias and HU-stability, which we use in this system. For more information on stability, we recommend reading these papers.[33–36] We introduce some essential contributions of researchers for many new concepts and ideas to the Caputo–Fabrizio derivative and ABC-derivative. For example, Li and Gou[37] discussed the existence of mild solutions for fractional impulsive evolution equations, including non-compact semigroup and periodic boundary conditions. Wang *et al.*[38] studied the existence of a PC-mild solution for impulsive FEDs with non-local conditions. To provide monotone sequential approximations for solutions to the initial value problems, Agarwal *et al.*[39] utilized lower and higher solutions and monotonicity concepts. Baleanu *et al.*[40] used the cosine family theory to investigate the existence and approximate controllability of mild solutions to impulsive FEDs. Shah *et al.*[41] studied existence of a solution to a class of implicit FDEs, including some impulsive and initial conditions. Dabas and Chauhan[42] investigated the possibility of a continuous mild solution for impulsive neutral FDEs with indefinite delay. For further details, it can be found in Refs. 43–48. Liu *et al.*[49] studied EU of solutions for an impulsive FDEs with p-Laplacian as follows:

$$\begin{cases} \mathcal{D}^{\lambda^*}_{0^+}\phi_p(\mathcal{D}^{\delta^*}_{0^+}\mu(\zeta)) = \mathcal{Y}^*_1(\zeta,\mu(\zeta)), \quad \zeta \in \mathcal{T}', \\ \omega\mu(\zeta_k) = \mathcal{I}_k(\mu(\zeta_k)), \omega\phi_p(\mathcal{D}^{\delta^*}_{0^+}\mu(\zeta_k)) = c_k, \\ \qquad\qquad k = 1,2,\ldots,m, \\ \mu(0) = \mu_0, \quad \mathcal{D}^{\delta^*}\mu(\zeta)|_{\zeta=0} = \mu_1, \end{cases}$$

where $\mathcal{D}^{\lambda^*}_{0^+}, \mathcal{D}^{\delta^*}_{0^+}$ are Caputo derivatives of order λ^* and δ^* where $0 < \lambda^*, \delta^* \leq 1, \lambda^* + \delta^* \leq 2$. $\mu_0, \mu_1 \in \mathbb{R}, \mathcal{T} = [0,1], \zeta_j \leq \zeta_{j+1}$ for $j = 0,1,\ldots,m$ with $\zeta_{m+1} = 1, \mathcal{T}' = \mathcal{T}\backslash\{\zeta_1,\ldots,\zeta_m\}$. $\omega\nu(\zeta_k) = \mu(\zeta_k^+) - \mu(\zeta_k^-)$ where $\mu(\zeta_k^-), \mu(\zeta_k^+)$ are the left and right limits of $\mu(\zeta)$ at $\zeta = \zeta_k (k = 1,2,\ldots,m)$, respectively. $\mathcal{Q}_1(\zeta,\mu(\zeta)), \mathcal{I}_k(\mu(\zeta_k))$ are continuous functions. By using fixed-point methods, Feckan *et al.*[50] discovered certain adequate conditions for the existence of solutions

$$\begin{cases} \mathcal{D}^{\lambda^*}_{0^+}\kappa(\zeta) = \mathcal{K}_1(\zeta,\kappa(\zeta)), \qquad \zeta \in \mathcal{T}', \\ \kappa(0) = \kappa_0, \quad \omega\kappa(\zeta_j) = \mathcal{I}_j(\kappa(\zeta_j^-)), \quad j = 1,2,\ldots,n, \end{cases}$$

where $\mathcal{D}^{\lambda^*}_{0^+}$ is Caputo factional derivative of order $0 < \lambda^* < 1$, $\mathcal{K}_1(\zeta,\kappa(\zeta))$ is continuous function $\mathcal{I}_j : \mathbb{R} \to \mathbb{R}$. $\mu_0 \in \mathbb{R}, \mathcal{T} = [0,1], \zeta_i \leq \zeta_{i+1}$ for $i = 0,\ldots,n$, with $\zeta_{n+1} = 1, \mathcal{T}' = \mathcal{T}\backslash\{\zeta_1,\ldots,\zeta_n\}$ are the right and left limits of $\kappa(\zeta)$ at $\zeta = \zeta_j (j = 1,2,\ldots,n)$. Inspired by the studies aforementioned, it has not been considered until yet in impulsive FDEs involving ABC-derivative and Caputo–Fabrizio's derivatives. In this literature, we investigate the existence and uniqueness of the solution for impulsive FDEs involving ABC-derivative and Caputo–Fabrizio derivatives with a p-Laplacian operator, as well we examine the HU-stability of the solution given by

$$\begin{cases} {}^{\mathrm{ABC}}_{0}\mathcal{D}^{\lambda_1}(\phi_p({}^{\mathrm{CF}}_{0}\mathcal{D}^{\sigma_1}(\mu_1(\zeta))) = \mathcal{Y}^*_1(\zeta,\mu_1(\zeta)), \\ \qquad\qquad \zeta \in \aleph = [0,\mathcal{T}], \\ \Delta[\phi_p({}^{\mathrm{CF}}_{0}\mathcal{D}^{\sigma_1}(\mu_1(\zeta_k)))] \\ \qquad = \mu_1(\zeta_k^+) - \mathcal{I}_k(\zeta_k^-) = \mu_1(\zeta_k^-), \\ \Delta\mu_1(\zeta_k) = \mu_1(\zeta_k^+) - \mathcal{I}_k(\zeta_k^-) = \mu_1(\zeta_k^-), \\ [\phi_p({}^{\mathrm{CF}}_{0}\mathcal{D}^{\sigma_1}(\mu_1(0)))] = \psi_0, \quad \mu_1(0) = \mu_0, \end{cases} \tag{1.1}$$

where ${}^{\mathrm{ABC}}_{0}\mathcal{D}^{\lambda_1}$ is ABC-fractional of orderλ_1 and ${}^{\mathrm{CF}}_{0}\mathcal{D}^{\sigma_1}$ is Caputo–Fabrizio derivative of order σ_1, $0 < \lambda_1, \sigma_1 \leq 1$ and $\mathcal{Y}^*_1 : \aleph \times \mathbb{R} \to \mathbb{R}^n$ is continuous function. Further $\mathcal{Y}^*_1(0,\mu_1(0)) = 0$, for all $\mu_1 > 0$, $\psi^* : PC(\aleph,\mathbb{R}) \to \mathbb{R}^n$, $\mu_\zeta(s) = \mu(\zeta + s)$ for $-\varsigma \leq s \leq 0$, $\mu_1(\zeta_k^+) = \lim_{\epsilon\to 0^+}\mu_1(\zeta_k + \epsilon)$ while $\mu_1(\zeta_k^-) = \lim_{\epsilon\to 0^-}\mu_1(\zeta_k + \epsilon)$, $0 =< \zeta_0 < \zeta_1 < \zeta_2 < \cdots < \zeta_m < \zeta_{m+1} = \mathcal{T}, \mathcal{I}_k : \mathbb{R}^n \to \mathbb{R}^n$ are continuous functions for $k = 1,2,\ldots,m$ and, respectively, $\mu_1(\zeta_k^+), \mu_1(\zeta_k^-)$ are the left-hand and the right-hand limits of the function $\mu_1(\zeta)$ at $\zeta = \zeta_k$. The $\phi_p(\kappa) = \kappa|\kappa|^{(p-2)}$ is the p-Laplacian operator and $\phi_p^{-1} = \phi_q$, such that $1/p + 1/q = 1$. Our proposed problem impulsive FDEs involving ABC-fractional equations and Caputo–Fabrizio's derivative is more complex and general than those mentioned above. To investigate the existence of the solution and stability of our suggested problem, we shall turn our suggested problem into an equivalent integral using the fixed-point theory ${}^{\mathrm{CF}}_{0}\mathcal{I}^{\sigma_1}$, ${}^{\mathrm{ABC}}_{0}\mathcal{I}^{\lambda_1}$. We shall provide an interpretive example for the application.

2. PRELIMINARIES

We present notations, definitions desired, and some background information in this section, which are necessary for this paper.

Consider space $\mathrm{PC}(\aleph, R) = \{\mu_1 : \aleph \to R : \mu_1 \in C(\zeta_j,\zeta_{j+1}], R; j = 1,\ldots,n+1$, and there exist $\mu_1(\zeta_j^+)$ and $\mu_1(\zeta_j^-), j = 1,\ldots,n$, with $\mu_1(\zeta_j^+), \mu_1(\zeta_j^-)\}$. The space $\mathrm{PC}(\aleph, R)$ is a Banach space with the norm $\|\mu_1\|_{\mathrm{PC}} = \max_{\zeta\in\aleph}|\mu_1(\zeta)|$. Set $\aleph = [0,\mathcal{T}]$ and $\aleph' = \aleph\backslash\{\zeta_j,\ldots,\zeta_j\}$.

Definition 1 (Refs. 51 and 9). The ABC-fractional derivative of the function $\mathcal{K} \in \mathcal{H}^*(c^*,d^*)$, and $d^* > c^*, 0 \leq \gamma^* \leq 1$ is defined as

$$\begin{aligned} &{}^{\mathrm{ABC}}_{c^*}\mathcal{D}^{\gamma^*}_{\zeta}\mathcal{K}(\zeta) \\ &\quad = \frac{\mathbb{N}(\gamma^*)}{1-\gamma^*}\int_{c^*}^{\zeta}\mathcal{K}'(\theta_0)\mathbb{E}_{\gamma^*}\left[\frac{-\gamma^*(\zeta-\theta_0)^{\gamma^*}}{1-\gamma^*}\right]d\theta_0, \end{aligned}$$

and

$$\begin{aligned} &{}^{\mathrm{ABC}}\mathcal{D}^{\gamma^*}_{d^*}\mathcal{K}(\zeta) \\ &\quad = \frac{\mathbb{N}(\gamma^*)}{1-\gamma^*}\int_{\zeta}^{d^*}\mathcal{K}'(\theta_0)\mathbb{E}_{\gamma^*}\left[\frac{-\gamma^*(\theta_0-\zeta)^{\gamma^*}}{1-\gamma^*}\right]d\theta_0, \end{aligned}$$

where $\mathbb{N}(\gamma^*) = 1 - \gamma^* + \dfrac{\gamma^*}{\Gamma(\gamma^*)}$, $\mathbb{N}(0) = \mathbb{N}(1) = 1$. $\mathbb{N}(\gamma^*)$ is the normalization function.

Definition 2 (Ref. 51). The ABR-fractional derivative of the function $\mathcal{K} \in \mathcal{H}^*(c^*, d^*)$, and $d^* > c^*, 0 \le \gamma^* \le 1$ is defined as

$$ {}^{\mathrm{ABR}}_{c^*}\mathcal{D}^{\gamma^*}_{\zeta}\mathcal{K}(\zeta) = \frac{\mathbb{N}(\gamma^*)}{1-\gamma^*}\frac{d}{d\zeta}\int_{c^*}^{\zeta}\mathcal{K}(\theta_0)\mathbb{E}_{\epsilon^*}\left[\frac{-\gamma^*(\zeta-s)^{\gamma^*}}{1-\gamma^*}\right]d\theta_0, $$

and

$$ {}^{\mathrm{ABR}}\mathcal{D}^{\gamma^*}_{d^*}\mathcal{K}(\zeta) = \frac{\mathbb{N}(\gamma^*)}{1-\gamma^*}\frac{-d}{d\zeta}\int_{\zeta}^{d^*}\mathcal{K}(\theta_0)\mathbb{E}_{\gamma^*}\left[\frac{-\gamma^*(\zeta-s)^{\gamma^*}}{1-\gamma^*}\right]d\theta_0. $$

Definition 3 (Ref. 52). The AB-fractional integral of the function $\mathcal{K} \in \mathcal{H}^*(c^*, d^*)$, $d^* > c^*, \gamma^* \in [0, 1]$ is defined as

$$ {}^{\mathrm{AB}}_{0}\mathcal{I}^{\gamma^*}_{\zeta}\mathcal{K}(\zeta) = \frac{1-\gamma^*}{\mathbb{N}(\gamma^*)}\mathcal{K}(\zeta) + \frac{\gamma^*}{\mathbb{N}(\gamma^*)\Gamma(\gamma^*)} \times \int_{c^*}^{\zeta}\mathcal{K}(\theta_0)(\zeta-\theta_0)^{\gamma^*-1}d\theta_0, $$

and

$$ {}^{\mathrm{AB}}\mathcal{I}^{\gamma^*}_{\zeta}\mathcal{K}(\zeta) = \frac{1-\gamma^*}{\mathbb{N}(\gamma^*)}\mathcal{K}(\zeta) + \frac{\gamma^*}{\mathbb{N}(\gamma^*)\Gamma(\gamma^*)} \times \int_{\zeta}^{d^*}\mathcal{N}(\theta_0)(\theta_0-\zeta)^{\gamma^*-1}d\theta_0. $$

Lemma 1 (Refs. 52 and 53). *Let* $\mathcal{K} \in \mathcal{H}^*(c^*, d^*)$, $d^* > c^*, 0 \le \gamma^* \le 1$, *we obtain*

$$ {}^{\mathrm{AB}}_{c^*}\mathcal{I}^{\gamma^*}({}^{\mathrm{ABC}}_{c^*}\mathcal{D}^{\gamma^*}_{\zeta})\mathcal{K}(\zeta) = \mathcal{K}(\zeta) - \mathcal{K}(c^*), $$

and

$$ {}^{\mathrm{AB}}\mathcal{I}^{\gamma^*}_{d^*}({}^{\mathrm{ABC}}\mathcal{D}^{\gamma^*}_{d^*})\mathcal{K}(\zeta) = \mathcal{K}(\zeta) - \mathcal{K}(d^*). $$

Definition 4 (Ref. 51). Let $\varpi > 0, \mathcal{Y}_1^* \in \mathcal{H}^*(0, \varpi)$, and $0 < \sigma_1 < 1$; then the σ_1th-order Caputo–Fabrizio derivative of $\mathcal{Y}_1^*$ is defined as

$$ {}^{\mathrm{CF}}_{0}\mathcal{D}^{\sigma_1}_{\zeta}\mathcal{Y}_1^*(\zeta) = \frac{(2-\sigma_1)\mathbb{W}(\sigma_1)}{2(1-\sigma_1)}\int_0^{\zeta}\frac{d(\mathcal{Y}_1^*(\eta))}{d\eta} \times \exp\left[-\sigma_1\frac{\zeta-\eta}{1-\sigma_1}\right]d\eta, $$

where $\mathbb{M}(\sigma_1)$ is a normalizing function. Furthermore, $\mathbb{W}(0) = \mathbb{W}(1) = 1$. In Ref. 54, Losada and Nieto presented the formula for $\mathbb{W}(\sigma_1)$ as $\mathbb{W}(\sigma_1) = \frac{2}{2-\sigma_1}$, $0 \le \sigma_1 \le 1$. The Caputo–Fabrizio derivative is simplified to formula

$$ {}^{\mathrm{CF}}_{0}\mathcal{D}^{\sigma_1}_{\zeta}\mathcal{Y}_1^*(\zeta) = \frac{1}{1-\sigma_1}\int_0^{\zeta}\frac{d(\mathcal{Y}_1^*(\eta))}{d\eta} \times \exp\left[-\sigma_1\frac{\zeta-\eta}{1-\sigma_1}\right]d\eta. $$

A function $\mathcal{Y}_1^*$ of order $\gamma^* > 0$ has Riemann–Liouville integral, $\mathcal{Y}_1^* : (0, +\infty) \to \mathcal{R}$, is defined by

$$ {}_{0}\mathcal{I}^{\gamma^*}\mathcal{Y}_1^*(\zeta) = \frac{1}{\Gamma(\gamma^*)}\int_0^{\zeta}(\zeta-s)^{\gamma^*-1}\mathcal{Y}_1^*(s)ds, $$

where $\mathrm{Re}(\gamma^*) > 0$ we have

$$ \Gamma(\gamma^*) = \int_0^{+\infty} e^{-s}s^{\gamma^*-1}ds. $$

The Caputo–Fabrizio integral of σ_1th-order is given by[54]

$$ {}^{\mathrm{CF}}\mathcal{I}^{\sigma_1}_{\zeta}\mathcal{Y}_1^*(\zeta) = \frac{2(1-\sigma_1)}{2\mathbb{W}(\sigma_1)-\sigma_1\mathbb{W}(\sigma_1)}\mathcal{Y}_1^*(\zeta) + \frac{2\sigma_1}{2\mathbb{W}(\sigma_1)-\sigma_1\mathbb{W}(\sigma_1)}\int_0^{\zeta}\mathcal{Y}_1^*(s)ds, \quad \zeta \ge 0, $$

where $\frac{2(1-\sigma_1)}{(2-\sigma_1\mathbb{W}(\sigma_1))} + \frac{2\sigma_1}{(2-\sigma_1)\mathbb{W}(\sigma_1)} = 1$.

Definition 5 (Ref. 55). Let $\mho$ be a Banach space. Then the operator $\Im : \mho \to \mho$ is a contraction if $\|\Im(\hbar_1) - \Im(\hbar_2)\| \le \ell\|\hbar_1\hbar_2\|$ for all $\hbar_1, \hbar_2 \in \mho, 0 < \ell < 1$.

Theorem 1 (Ref. 55). *Let* $\mho$ *be a Banach space and* ψ *be a non-empty closed subset of* $\mho$. *If* $\Im : \psi \to \psi$ *is a contraction, then there exists a unique fixed point of* $\Im$.

Theorem 2 (Ref. 55). *Let* $\mho$ *be a Banach space and let* $\Im : \mho \to \mho$ *be a continuous and compact mapping. Moreover, suppose* $\omega = \{\mu_1 \in \mho : \mu_1 = \gamma\Im\mu_1,$ *for some* $\gamma \in (0, 1)\}$, *be a bounded set. Then* $\Im$ *has at least one fixed point in* $\mho$.

Lemma 2 (Ref. 56). *Let* ϕ_q^* *be p-Laplacian. Then we have*

(ξ_1) *If* $1 < q^* \le 2, \varsigma_1, \varsigma_2 > 0$ *and* $|\varsigma_1|, |\varsigma_2| \ge \rho_1^* > 0$, *then*

$$ |\phi_q^*(\varsigma_1) - \phi_q^*(\varsigma_2)| \le (q^*-1)\rho_1^{*(q^*-2)}|\varsigma_1 - \varsigma_2|. $$

(ξ_2) *If* $q^* > 2$ *and* $|\varsigma_1|, |\varsigma_2| \le \rho_2^*$, *then*

$$ |\phi_q^*(\varsigma_1) - \phi_q^*(\varsigma_2)| \le (q^*-1)\rho_2^{*(q^*-2)}|\varsigma_1 - \varsigma_2|. $$

3. EXISTENCE AND UNIQUENESS

Theorem 3. *For $\lambda_1, \sigma_1 \in (0,1]$, a function $\mathcal{Y}_1^*(\zeta, \mu_1(\zeta)) \in C[0,1]$ satisfies* (1.1). *Then the solution of impulsive FDEs with a p-Laplacian operator*:

$$\begin{cases} {}_0^{\mathrm{ABC}}\mathcal{D}^{\lambda_1}(\phi_p({}_0^{\mathrm{CF}}\mathcal{D}^{\sigma_1}(\mu_1(\zeta))) = \mathcal{Y}_1^*(\zeta, \mu_1(\zeta)), \quad \zeta \in \aleph = [0, \mathcal{T}], \\ \Delta[\phi_p({}_0^{\mathrm{CF}}\mathcal{D}^{\sigma_1}(\mu_1(\zeta_k)))] = \mu_1(\zeta_k^+) - \mathcal{I}_k(\zeta_k^-) = \mu_1(\zeta_k^-), \\ \Delta\mu_1(\zeta_k) = \mu_1(\zeta_k^+) - \mathcal{I}_k(\zeta_k^-) = \mu_1(\zeta_k^-), \\ [\phi_p({}_0^{\mathrm{CF}}\mathcal{D}^{\sigma_1}(\mu_1(0)))] = \psi_0, \quad \mu(0) = \mu_0, \end{cases} \tag{3.1}$$

is given by integral equation

$$\mu_1(\zeta) = \begin{cases} \psi_0 + \mathbb{A}_1\phi_q(\mathbb{G}(\mu_1(\zeta))) + \mathbb{A}_2\displaystyle\int_0^\zeta (\zeta-\tau)^{\sigma_1-1}(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau, & \text{for } \zeta \in [0,\zeta_1], \\ \psi_0 + \mathbb{A}_1[\mathbb{G}(\mu_1(\zeta_1)) + \mathbb{G}(\mu_1(\zeta))] + \mathbb{A}_2\left[\displaystyle\int_0^{\zeta_1} (\zeta_1-\tau)^{\sigma_1-1}(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau \right. \\ \quad \left. + \displaystyle\int_{\zeta_1}^{\zeta} (\zeta-\tau)^{\sigma_1-1}(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau\right] + \mathcal{I}_1\mu_1(\zeta_1^-), & \text{for } \zeta \in [\zeta_1,\zeta_2], \\ \vdots \\ \psi_0 + \mathbb{A}_1\displaystyle\sum_{k=1}^{m+1}\mathbb{G}(\mu_1(\zeta_k)) + \mathbb{A}_2\left[\displaystyle\sum_{k=1}^{m}\int_{\zeta_{k-1}}^{\zeta_k} (\zeta_k-\tau)^{\sigma_1-1}(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau \right. \\ \quad \left. + \displaystyle\int_{\zeta_m}^{\zeta} (\zeta-\tau)^{\sigma_1-1} \times (\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau\right] + \displaystyle\sum_{k=1}^{m}\mathcal{I}_k\mu_1(\zeta_k^-), & \text{for } \zeta \in [\zeta_m, \mathcal{T}], \end{cases} \tag{3.2}$$

where

$$\begin{aligned} \mathbb{G}(\mu_1(\zeta)) = \psi_0 &+ \sum_{k=1}^{m+1}(\mathcal{Y}_1^*(\zeta_k, \mu_1(\zeta_k)) \\ &+ \sum_{k=1}^{m}\mathcal{I}_k\mu_1(\zeta_k^-) \\ &+ \sum_{k=1}^{m}\left[\int_{\zeta_{k-1}}^{k}(\zeta_k - s)^{\lambda_1-1}\mathcal{Y}_1^*(s, \mu_1(s))ds \right. \\ &\left. + \int_{\zeta_m}^{\zeta}(\zeta - s)^{\lambda_1-1}\mathcal{Y}_1^*(s, \mu_1(s))ds\right], \end{aligned} \tag{3.3}$$

where

$$\mathbb{A}_1 = \frac{2(1-\sigma_1)}{2\mathbb{M}(\sigma_1) - \sigma_1\mathbb{M}(\sigma_1)},$$

$$\mathbb{A}_2 = \frac{2\sigma_1}{2\mathbb{M}(\sigma_1) - \sigma_1\mathbb{M}(\sigma_1)},$$

$$\mathbb{B}_1 = \frac{1-\lambda_1}{\mathbb{B}(\lambda_1)},$$

$$\mathbb{B}_2 = \frac{\lambda_1}{\mathbb{B}(\lambda_1)\Gamma(\lambda_1)}.$$

Here, we define some hypotheses to prove the existence and uniqueness of solution for our system, as follows:

($\mathbb{H}_1$) Let us assume for $\mu_1(\tau), \mu_1(s) \in PC(\aleph, \mathbb{R})$ there exist constants $\chi^*, \kappa^* \in (0,1)$ such that

$$\|\mathcal{Y}_1^*(\tau, \mu_1(\tau))\| \le \kappa^*,$$
$$\|\mathcal{Y}_1^*(s, \mu_1(s))\| \le \chi^*, \quad \text{for } \tau, s \in [0,1].$$

($\mathbb{H}_2$) Let there for $\mu_1, \bar{z}_1 \in PC(\aleph, \mathbb{R})$ we have some constants $\mathcal{K}, \mathcal{L} \in (0,1)$ for each $\zeta, s \in [0,1]$, such that

$$\|\mathcal{Y}_1^*(\zeta,\mu_1(\zeta))-\mathcal{Y}_1^*(\zeta,\bar{z}_1(\zeta))\| \le \mathcal{K}\|\mu_1-\bar{z}_1\|,$$

$$\|\mathcal{Y}_1^*(s,\mu_1(s))-\mathcal{Y}_1^*(s,\bar{z}_1(s))\| \le \mathcal{L}\|\mu_1-\bar{z}_1\|.$$

($\mathbb{H}_3$) Let the impulses $\mathcal{I}_k \in C(\mathbb{R}^n,\mathbb{R}^n)$ be bounded and for each $\mu_1, z_1 \in PC(\aleph,\mathbb{R})$ have some constants $\mathcal{O},\Delta_1 \in (0,1)$ for $k=1,2,\ldots,m$, for $\zeta\in[0,1]$, such that

$$\|\mathcal{I}_k^*\mu_1(\zeta_k^-)-\mathcal{I}_k^*(z_1(\zeta_k^-))\| \le \mathcal{O}\|\mu_1-z_1\|,$$
$$\|\mathcal{I}_k^*\mu_1(\zeta_k^-)\| \le \Delta_1.$$

Define the operator $\mathcal{F}:\xi\to\xi$ by

$$\mathcal{F}\mu_1(\zeta)=\begin{cases}\psi_0+\mathbb{A}_1\phi_q(\mathbb{G}(\mu_1(\zeta)))+\mathbb{A}_2\displaystyle\int_0^{\zeta}(\zeta-\tau)^{\sigma_1-1}(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau, & \text{for } \zeta\in[0,\zeta_1],\\ \psi_0+\mathbb{A}_1[\mathbb{G}(\mu_1(\zeta_1))+\mathbb{G}(\mu_1(\zeta))]+\mathbb{A}_2\Bigg[\displaystyle\int_0^{\zeta_1}(\zeta_1-\tau)^{\sigma_1-1}(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau \\ \quad+\displaystyle\int_{\zeta_1}^{\zeta}(\zeta-\tau)^{\sigma_1-1}(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau\Bigg]+\mathcal{I}_1\mu_1(\zeta_1^-), & \text{for } \zeta\in[\zeta_1,\zeta_2],\\ \vdots \\ \psi_0+\mathbb{A}_1\displaystyle\sum_{k=1}^{m+1}\mathbb{G}(\mu_1(\zeta_k))+\mathbb{A}_2\Bigg[\sum_{k=1}^{m}\int_{\zeta_{k-1}}^{\zeta_k}(\zeta_k-\tau)^{\sigma_1-1}(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau \\ \quad+\displaystyle\int_{\zeta_m}^{\zeta}(\zeta-\tau)^{\sigma_1-1}(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau\Bigg]+\sum_{k=1}^{m}\mathcal{I}_k\mu_1(\zeta_k^-), & \text{for } \zeta\in[\zeta_m,\mathcal{T}].\end{cases} \tag{3.4}$$

Theorem 4. *Presume that the conditions* ($\mathbb{H}_1$) − ($\mathbb{H}_3$) *hold, the impulsive system* (1.1) *has at least one solution where the following inequality is satisfied*:

$$\phi_q[\mathbb{B}_1(m+1)\kappa^*+\Delta_1 m+\mathbb{B}_2(m+1)\chi^*] \times\left[\mathbb{A}_1(m+1)+\frac{\mathbb{A}_2(m+1)}{\Gamma(\sigma_1)}\right]+\Delta_1 m\le 1.$$

Proof. By the Banach space of $PC([\aleph,\mathcal{T}])$, we have that $\mathrm{PC}([\aleph,\mathcal{T}])$ which is convex, bounded, and closed subset of $\mathrm{PC}([\aleph,\mathcal{T}],\mathbb{R}^n)$. For the existence of a solution, we use Schauder's fixed-point theory. For that, we split the proof into the following steps.

Step 1. Here, we will prove that $\mathcal{F}:PC([\aleph,\mathcal{T}])\to PC([\aleph,\mathcal{T}])$ convex, bounded, and closed. At first, for $\zeta\in[0,\zeta_1]$, (3.4) proceed to

$$\begin{aligned}\|\mathcal{F}\mu_1(\zeta)\| &= \|\mathbb{A}_1\phi_q(\mathbb{G}(\mu_1(\zeta)))+\mathbb{A}_2\int_0^{\zeta}(\zeta-\tau)^{\sigma_1-1} \times(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau\| \\ &\le \mathbb{A}_1\phi_q\Bigg[\mathbb{B}_1\|\mathcal{Y}_1^*(\tau,\mu_1(\tau))\| +\mathbb{B}_2\int_0^{\tau}(\tau-s)^{\lambda_1-1}\|\mathcal{Y}_1^*(s,\mu_1(s))\|ds\Bigg] \\ &\quad+\mathbb{A}_2\int_0^{\zeta}(\zeta-\tau)^{\sigma_1-1}\phi_q \times\Bigg[\mathbb{B}_1\|\mathcal{Y}_1^*(\tau,\mu_1(\tau))\| +\mathbb{B}_2\int_0^{\tau}(\tau-s)^{\lambda_1-1}\|\mathcal{Y}_1^*(s,\mu_1(s)))\|ds\Bigg] \\ &\le \phi_q(\mathbb{B}_1\kappa^*+\mathbb{B}_2\chi^*)\left[\mathbb{A}_1+\frac{\mathbb{A}_2}{\Gamma(\sigma_1)}\right]\le 1.\end{aligned} \tag{3.5}$$

Now, for the $\zeta\in(\zeta_k,\zeta_{k+1}]$, we have

$$\|\mathcal{F}\mu_1(\zeta)\| = \Bigg\|\mathbb{A}_1\phi_q\sum_{k=1}^{m+1}\mathbb{G}(\mu_1(\zeta_k)) +\mathbb{A}_2\Bigg[\sum_{k=1}^{m}\int_{\zeta_{k-1}}^{\zeta_k}(\zeta_k-\tau)^{\sigma_1-1}$$

$$\times(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau + \int_{\zeta_m}^{\zeta}(\zeta-\tau)^{\sigma_1-1}$$
$$\left.\left.\times(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau\right] + \sum_{k=1}^{m}\mathcal{I}_k\mu_1(\zeta_k^-)\right\|$$
$$\leq \mathbb{A}_1\phi_q\sum_{k=1}^{m+1}\left[\mathbb{B}_1\sum_{k=1}^{m+1}\|\mathcal{Y}_1^*(\tau_k,\mu_1(\tau_k))\|\right.$$
$$+\sum_{k=1}^{m}\|\mathcal{I}_k\mu_1(\tau_k^-)\|$$
$$+\mathbb{B}_2\sum_{k=1}^{m}\int_{\tau_{k-1}}^{\tau_k}(\tau_k-s)^{\lambda_1-1}$$
$$\times\|\mathcal{Y}_1^*(s,\mu_1(s))\|ds+\mathbb{B}_1$$
$$\left.\times\int_{\tau_m}^{\tau}(\tau-s)^{\lambda_1-1}\|\mathcal{Y}_1^*(s,\mu_1(s))\|ds\right]$$
$$+\mathbb{A}_2\left[\sum_{k=1}^{m}\int_{\zeta_{k-1}}^{\zeta_k}(\zeta_k-\tau)^{\sigma_1-1}\right.$$
$$\times\left(\phi_q\left[\mathbb{B}_2\sum_{k=1}^{m+1}\|\mathcal{Y}_1^*(\tau_k,\mu_1(\tau_k))\|\right.\right.$$
$$+\sum_{k=1}^{m}\|\mathcal{I}_k\mu_1(\tau_k^-)\|$$
$$+\mathbb{B}_1\sum_{k=1}^{m}\int_{\tau_{k-1}}^{k}(\tau_k-s)^{\lambda_1-1}$$
$$\times\|\mathcal{Y}_1^*(s,\mu_1(s))\|ds+\mathbb{B}_1\int_{\tau_m}^{\tau}(\tau-s)^{\lambda_1-1}$$
$$\left.\left.\times\|\mathcal{Y}_1^*(s,\mu_1(s))\|ds\right]d\tau\right)$$
$$+\int_{\zeta_m}^{\zeta}(\zeta-\tau)^{\sigma_1-1}$$
$$\times\left(\phi_q\left[\mathbb{B}_2\sum_{k=1}^{m+1}\|\mathcal{Y}_1^*(\tau_k,\mu_1(\tau_k))\|\right.\right.$$
$$+\sum_{k=1}^{m}\|\mathcal{I}_k\mu_1(\tau_k^-)\|$$
$$+\mathbb{B}_1\sum_{k=1}^{m}\int_{\tau_{k-1}}^{\tau_k}(\tau_k-s)^{\lambda_1-1}$$
$$\times\|\mathcal{Y}_1^*(s,\mu_1(s))\|ds$$
$$+\mathbb{B}_1\int_{\tau_m}^{\tau}(\tau-s)^{\lambda_1-1}$$
$$\left.\left.\left.\times\|\mathcal{Y}_1^*(s,\mu_1(s))\|ds\right]d\tau\right)\right]$$
$$+\sum_{k=1}^{m}\|\mathcal{I}_k\mu_1(\zeta_k^-)\|$$
$$\leq\phi_q[\mathbb{B}_1(m+1)\kappa^*+\Delta_1 m$$
$$+\mathbb{B}_2(m+1)\chi^*]$$
$$\times\left[\mathbb{A}_1(m+1)+\frac{\mathbb{A}_2(m+1)}{\Gamma(\sigma_1)}\right]$$
$$+\Delta_1 m\leq 1. \tag{3.6}$$

With the help of (3.5) and (3.6), we have $\mathcal{F}\mu_1(\zeta)\leq 1$. The first step is completed.

Step 2. Here, we will prove that $\mathcal{F}$ a continuous operator. For this, consider a convergent sequence $\mu_{1,n}\to\mu_1$ on the $PC([\aleph,\mathcal{T}])$. Since $\mathcal{Y}_1^*(\zeta,\mu_1(\zeta))$ and the impulses $\mathcal{I}_k^*\mu_1(\zeta_k^-)$ are continuous operators on $PC([\aleph,\mathcal{T}])$ for $k=1,2,\ldots,m$. Then for $\zeta\in[0,\zeta_1]$, we have

$$\|\mathcal{F}\mu_{1,n}(\zeta)-\mathcal{F}\mu_1(\zeta)\|$$
$$\leq\mathbb{A}_1\phi_q\|(\mathbb{G}(\mu_{1,n}(\zeta)))-\mathbb{G}(\mu_1(\zeta)))\|$$
$$+\mathbb{A}_2\int_0^{\zeta}(\zeta-\tau)^{\sigma_1-1}(\phi_q\|(\mathbb{G}(\mu_{1,n}(\tau))))$$
$$-(\mathbb{G}(\mu_1(\tau))))\|d\tau$$
$$\leq\mathbb{A}_1\phi_q[\mathbb{B}_1\|\mathcal{Y}_1^*(\zeta,\mu_{1,n}(\zeta))-\mathcal{Y}_1^*(\zeta,\mu_1(\zeta))\|$$
$$+\mathbb{B}_2\int_0^{\tau}(\tau-s)^{\lambda_1-1}\|\mathcal{Y}_1^*(s,\mu_{1,n}(s))$$
$$-\mathcal{Y}_1^*(s,\mu_1(s))\|ds+\frac{\mathbb{A}_2\zeta^{\sigma_1}\phi_q}{\Gamma(\sigma_1)}$$
$$\times\left[\mathbb{B}_1\|\mathcal{Y}_1^*(\zeta,\mu_{1,n}(\zeta))-\mathcal{Y}_1^*(\zeta,\mu_1(\zeta))\|\right.$$
$$+\mathbb{B}_2\int_0^{\tau}(\tau-s)^{\lambda_1-1}\|\mathcal{Y}_1^*(s,\mu_{1,n}(s))$$
$$\left.-\mathcal{Y}_1^*(s,\mu_1(s))\|\right]ds$$

$$\leq \left[\mathbb{A}_1\phi_q + \frac{\mathbb{A}_2\zeta^{\sigma_1}\phi_q}{\Gamma(\sigma_1)}\right] \times [\mathbb{B}_1\|\mathcal{Y}_1^*(\zeta,\mu_{1,n}(\zeta)) - \mathcal{Y}_1^*(\zeta,\mu_1(\zeta))\| + \mathbb{B}_2\|\mathcal{Y}_1^*(s,\mu_{1,n}(s)) - \mathcal{Y}_1^*(\zeta,\mu_1(\zeta))\|] \to 0. \tag{3.7}$$

Similarly, for $\zeta \in [\zeta_k, \zeta_{k+1}]$, we have

$$\begin{aligned}
&\|\mathcal{F}\mu_{1,n}(\zeta) - \mathcal{F}\mu_1(\zeta)\| \\
&\leq \mathbb{A}_1(q-1)\rho_1^{*(q-2)} \times \sum_{k=1}^{m+1}\left[\mathbb{B}_1\sum_{k=1}^{m+1}\|\mathcal{Y}_1^*(\zeta_k,\mu_{1,n}(\zeta_k)) - \mathcal{Y}_1^*(\zeta_k,\mu_1(\zeta_k))\| + \sum_{k=1}^{m}\mathcal{I}_k \times \|\mu_{1,n}(\zeta_k^-) - \mu_1(\zeta_k^-)\|\right.\\
&\quad + \mathbb{B}_2\sum_{k=1}^{m}\int_{\zeta_{k-1}}^{\zeta_k}(\zeta_k - s)^{\lambda_1-1} \times \|\mathcal{Y}_1^*(s,\mu_{1,n}(s)) - \mathcal{Y}_1^*(s,\mu_1(s))\|ds \\
&\quad \left. + \mathbb{B}_2\int_{\zeta_m}^{\zeta}(\zeta_- s)^{\lambda_1-1} \times \|\mathcal{Y}_1^*(s,\mu_{1,n}(s)) - \mathcal{Y}_1^*(s,\mu_1(s))\|ds\right] \\
&\quad + \mathbb{A}_2(q-1)\rho_1^{*(q-2)}\left[\sum_{k=1}^{m}\int_{\zeta_{k-1}}^{\zeta_k}(\zeta_k-\tau)^{\sigma_1-1} \times \left[\mathbb{B}_1\sum_{k=1}^{m+1}\|\mathcal{Y}_1^*(\zeta_k,\mu_{1,n}(\zeta_k)) - \mathcal{Y}_1^*(\zeta_k,\mu_1(\zeta_k))\|\right.\right.\\
&\quad + \sum_{k=1}^{m}\mathcal{I}_k\|\mu_{1,n}(\zeta_k^-) - \mu_1(\zeta_k^-)\| + \mathbb{B}_2\sum_{k=1}^{m}\int_{\zeta_{k-1}}^{\zeta_k}(\zeta_k - s)^{\lambda_1-1} \\
&\quad \times \|\mathcal{Y}_1^*(s,\mu_{1,n}(s)) - \mathcal{Y}_1^*(s,\mu_1(s))\|ds - \mathcal{Y}_1^* \times (\zeta_- s)^{\lambda_1-1}\|\mathcal{Y}_1^*(s,\mu_{1,n}(s)) \\
&\quad \left.\left. - \mathcal{Y}_1^*(s,\mu_1(s))\|ds\right]\right] d\tau + \int_{\zeta_m}^{\zeta}(\zeta-\tau)^{\sigma_1-1} \\
&\quad \times \left[\mathbb{B}_1\sum_{k=1}^{m+1}\|\mathcal{Y}_1^*(\zeta_k,\mu_{1,n}(\zeta_k)) - \mathcal{Y}_1^*(\zeta_k,\mu_1(\zeta_k))\|\right. + \sum_{k=1}^{m}\mathcal{I}_k\|\mu_{1,n}(\zeta_k^-) - \mu_1(\zeta_k^-)\| \\
&\quad + \mathbb{B}_2\sum_{k=1}^{m}\int_{\zeta_{k-1}}^{\zeta_k}(\zeta_k - s)^{\lambda_1-1}\|\mathcal{Y}_1^*(s,\mu_{1,n}(s)) - \mathcal{Y}_1^*(s,\mu_1(s))\|ds \\
&\quad + \mathbb{B}_2\int_{\zeta_m}^{\zeta}(\zeta_- s)^{\lambda_1-1}\|\mathcal{Y}_1^*(s,\mu_{1,n}(s)) \left.\left. - \mathcal{Y}_1^*(s,\mu_1(s))\|ds\right]\right] d\tau \\
&\quad + \sum_{k=1}^{m}\mathcal{I}_k\|\mu_{1,n}(\zeta_k^-) - \mu_1(\zeta_k^-)\| \to 0. \qquad (3.8)
\end{aligned}$$

With the aid of (3.7) and (3.8), we obtained the operator $\mathcal{F} : \mathrm{PC}([\aleph,\mathcal{T}]) \to \mathrm{PC}([\aleph,\mathcal{T}])$ which is continuous.

Step 3. In this step, we will prove of equicontinuity of the operator $\mathcal{F} : PC([\aleph,\mathcal{T}]) \to PC([\aleph,\mathcal{T}])$. For this, we consider

$$\begin{aligned}
&\|\mathcal{F}\mu_1(\zeta_2) - \mathcal{F}\mu_1(\zeta_1)\| \\
&\leq \mathbb{A}_1\phi_q\|(\mathbb{G}(\mu_1(\zeta_2))) - (\mathbb{G}(\mu_1(\zeta_1)))\| + \mathbb{A}_2\left\|\int_0^{\zeta_2}(\zeta_2-\tau)^{\sigma_1-1} - \int_0^{\zeta_1}(\zeta_1-\tau)^{\sigma_1-1}\right\| \times \phi_q\|(\mathbb{G}(\mu_1(\tau)))\|d\tau \\
&\leq \mathbb{A}_1\phi_q\left[\mathbb{B}_1\|\mathcal{Y}_1^*(\zeta_2,\mu_1(\zeta_2)) - \mathcal{Y}_1^*(\zeta_1,\mu_1(\zeta_1))\| + \mathbb{B}_2\left\|\int_0^{\zeta_2}(\zeta_2 - s)^{\lambda_1-1} - \int_0^{\zeta_1}(\zeta_1-s)^{\lambda_1-1}\right\|\|\mathcal{Y}_1^*(s,\mu_1(s))\|ds\right] \\
&\quad + \frac{\mathbb{A}_2(\zeta_2-\zeta_1)}{\Gamma(\sigma_1)}\phi_q\left[\mathbb{B}_1\|\mathcal{Y}_1^*(\tau,\mu_1(\tau))\| + \mathbb{B}_2\left\|\int_0^{\tau}(\tau-s)^{\lambda_1-1}\right\|\|\mathcal{Y}_1^*(s,\mu_1(s))\|ds\right] \\
&\leq \mathbb{A}_1\phi_q[\mathbb{B}_1\|\mathcal{Y}_1^*(\zeta_2,\mu_1(\zeta_2)) - \mathcal{Y}_1^*(\zeta_1,\mu_1(\zeta_1))\| + \mathbb{B}_2(\zeta_2-\zeta_1)\|\mathcal{Y}_1^*(s,\mu_1(s))\|]
\end{aligned}$$

$$+\mathbb{A}_2(\zeta_2-\zeta_1)\phi_q[\mathbb{B}_1\|\mathcal{Y}_1^*(\tau,\mu_1(\tau))\| + \mathbb{B}_2\|\mathcal{Y}_1^*(s,\mu_1(s))\|] \to 0. \quad (3.9)$$

With the aid of (3.9), we have $\|\mathcal{F}\mu_1(\zeta_2) - \mathcal{F}\mu_1(\zeta_1)\| \to 0$ as $\zeta_1 \to \zeta_2$. Similarly, we have for $\zeta \in (\zeta_k, \zeta_{k+1}]$. This means that the operator $\mathcal{F}$ is an equicontinuous operator. Thus, with the help of Arzela–Ascolli theory, $\mathcal{F}$ is relatively compact which implies $\mathcal{F}$ is completely continuous. Consequently, by Step 1 to Step 3, and Schauder's fixed-point theorem, the operator $\mathcal{F}$ has a fixed point. Thus, the impulsive system involving ABC-derivative and Caputo–Fabrizio derivative (1.1) has a solution. □

Theorem 5. *If the suppositions* $(\mathbb{H}_1)$–$(\mathbb{H}_3)$ *hold, the impulsive system with ABC-derivative and Caputo–Fabrizio derivative* (1.1) *has a unique solution provided that the following inequality is satisfied*:

$$\varpi^* = \left[\left[\mathbb{A}_1(m+1)(q-1)\rho_1^{*(q-2)} + \frac{\mathbb{A}_2(m+1)(q-1)\rho_1^{*(q-2)}}{\Gamma(\sigma_1)}\right]\left[\mathbb{B}_1(m+1)\mathcal{K} + \frac{\mathbb{B}_2(m+1)}{\lambda_1}\mathcal{L} + m\mathcal{O}\right] + m\mathcal{O}\right] < 1. \quad (3.10)$$

Proof. We suppose the contrary path for proving the uniqueness of solution that is let there exist two solutions say μ_1 and $\bar{\mu}_1$. Then we consider two cases

Case 1. Then for $\zeta \in [0, \zeta_1]$, we have

$$\begin{aligned}
&\|\mathcal{F}\mu_1(\zeta) - \mathcal{F}\bar{\mu}_1(\zeta)\| \\
&\quad \le \mathbb{A}_1\|(\phi_q\mathbb{G}(\mu_1(\zeta))) - (\phi_q\mathbb{G}(\bar{\mu}_1(\zeta)))\| \\
&\qquad + \mathbb{A}_2\int_0^\zeta (\zeta-\tau)^{\sigma_1-1}\|(\phi_q\mathbb{G}(\mu_1(\tau))) - (\phi_q\mathbb{G}(\bar{\mu}_1(\tau)))\|d\tau \\
&\quad \le \mathbb{A}_1(q-1)\rho_1^{*(q-2)}\Bigg[\mathbb{B}_1\|\mathcal{Y}_1^*(\zeta,\mu_1(\zeta)) - \mathcal{Y}_1^*(\zeta,\bar{\mu}_1(\zeta))\| + \mathbb{B}_2\left\|\int_0^\zeta (\zeta-s)^{\lambda_1-1}\right\| \\
&\qquad \times \|\mathcal{Y}_1^*(s,\mu_1(s)) - \mathcal{Y}_1^*(s,\bar{\mu}_1(s))\|ds\Bigg] \\
&\qquad + \frac{\mathbb{A}_2(q-1)\rho_1^{*(q-2)}}{\Gamma(\sigma_1)}\Bigg[\mathbb{B}_1\|\mathcal{Y}_1^*(\tau,\mu_1(\tau)) - \mathcal{Y}_1^*(\tau,\bar{\mu}_1(\tau))\| + \mathbb{B}_2\left\|\int_0^\zeta (\zeta-s)^{\lambda_1-1}\right\| \\
&\qquad \times \|\mathcal{Y}_1^*(s,\mu_1(s)) - \mathcal{Y}_1^*(s,\bar{\mu}_1(s))\|ds\Bigg] \\
&\quad \le \left[\left[\mathbb{A}_1(q-1)\rho^{q-2} + \frac{\mathbb{A}_2(q-1)\rho_1^{*(q-2)}}{\Gamma(\sigma_1)}\right] \times \left(\mathbb{B}_1\mathcal{K} + \frac{\mathbb{B}_2\mathcal{L}}{\lambda_1}\right)\right](\|\mu_1 - \bar{\mu}_1\|). \qquad (3.11)
\end{aligned}$$

Case 2. Let $\zeta \in (\zeta_k, \zeta_{k+1}]$. For this, consider

$$\begin{aligned}
&\|\mathcal{F}\mu_1(\zeta) - \mathcal{F}\bar{\mu}_1(\zeta)\| \\
&\quad \le \Bigg\|\mathbb{A}_1\sum_{k=1}^{m+1}(\phi_q\mathbb{G}(\mu_1(\zeta_k)) - \phi_q\mathbb{G}(\bar{\mu}_1(\zeta_k))) \\
&\qquad + \mathbb{A}_2\Bigg[\sum_{k=1}^{m}\int_{\zeta_{k-1}}^{\zeta_k}(\zeta_k-\tau)^{\sigma_1-1}(\phi_q(\mathbb{G}(\mu_1(\tau)) - \phi_q\mathbb{G}(\bar{\mu}_1(\tau))))d\tau + \int_{\zeta_m}^{\zeta}(\zeta-\tau)^{\sigma_1-1} \\
&\qquad \times (\phi_q(\mathbb{G}(\mu_1(\tau)) - \phi_q\mathbb{G}(\bar{\mu}_1(\tau))))d\tau\Bigg] \\
&\qquad + \sum_{k=1}^{m}(\mathcal{I}_k\mu_1(\zeta_k^-) - \mathcal{I}_k\bar{\mu}_1(\zeta_k^-))\Bigg\| \\
&\quad \le \mathbb{A}_1(q-1)\rho_1^{*(q-2)}(m+1) \\
&\qquad \times \Bigg[\mathbb{B}_1\sum_{k=1}^{m+1}\|\mathcal{Y}_1^*(\zeta_k,\mu_1(\zeta_k)) - \mathcal{Y}_1^*(\zeta_k,\bar{\mu}_1(\zeta_k))\| \\
&\qquad + \sum_{k=1}^{m}\|\mathcal{I}_k\mu_1(\zeta_k^-) - \mathcal{I}_k\bar{\mu}_1(\zeta_k^-)\| \\
&\qquad + \mathbb{B}_2\sum_{k=1}^{m}\int_{\zeta_{k-1}}^{\zeta_k}(\zeta_k-s)^{\lambda_1-1}\|\mathcal{Y}_1^*(s,\mu_1(s))
\end{aligned}$$

$$
\begin{aligned}
& - \mathcal{Y}_1^*(s, \bar{\mu}_1(s))\| ds + \mathbb{B}_2 \int_{\zeta_m}^{\zeta} (\zeta - s)^{\lambda_1 - 1} \\
& \times \|\mathcal{Y}_1^*(s, \mu_1(s)) - \mathcal{Y}_1^*(s, \bar{\mu}_1(s))\| ds \Bigg] \\
& + \mathbb{A}_2 \Bigg[\frac{(q-1)\rho_1^{*(q-2)} m}{\Gamma(\sigma_1)} \\
& \times \Bigg[\mathbb{B}_1 \sum_{k=1}^{m+1} \|\mathcal{Y}_1^*(\tau_k, \mu_1(\tau_k)) \\
& - \mathcal{Y}_1^*(\tau_k, \bar{\mu}_1(\tau_k))\| \\
& + \sum_{k=1}^{m} \|\mathcal{I}_k \mu_1(\tau_k^-) - \mathcal{I}_k \bar{\mu}_1(\tau_k^-)\| \\
& + \mathbb{B}_2 \sum_{k=1}^{m} \int_{\tau_{k-1}}^{\tau_k} (\tau_k - s)^{\lambda_1 - 1} \|\mathcal{Y}_1^*(s, \mu_1(s)) \\
& - \mathcal{Y}_1^*(s, \bar{\mu}_1(s))\| \Bigg] ds + \mathbb{B}_2 \int_{\tau_m}^{\tau} (\tau - s)^{\lambda_1 - 1} \\
& \times \|\mathcal{Y}_1^*(s, \mu_1(s)) - \mathcal{Y}_1^*(s, \bar{\mu}_1(s)) ds \Bigg] \\
& + \frac{(q-1)\rho_1^{*(q-2)}}{\Gamma(\sigma_1)} \Bigg[\mathbb{B}_1 \sum_{k=1}^{m+1} \|\mathcal{Y}_1^*(\tau_k, \mu_1(\tau_k)) \\
& - \mathcal{Y}_1^*(\tau_k, \bar{\mu}_1(\tau_k))\| + \sum_{k=1}^{m} \|\mathcal{I}_k \mu_1(\tau_k^-) \\
& - \mathcal{I}_k \bar{\mu}_1(\tau_k^-)\| + \mathbb{B}_2 \sum_{k=1}^{m} \int_{\tau_{k-1}}^{\tau_k} (\tau_k - s)^{\lambda_1 - 1} \\
& \times \|\mathcal{Y}_1^*(s, \mu_1(s)) - \mathcal{Y}_1^*(s, \bar{\mu}_1(s))\| \Bigg] ds \\
& + \mathbb{B}_2 \int_{\tau_m}^{\tau} (\tau - s)^{\lambda_1 - 1} \\
& \times \|\mathcal{Y}_1^*(s, \mu_1(s)) - \mathcal{Y}_1^*(s, \bar{\mu}_1(s))\| ds \Bigg]\Bigg] \\
& + \sum_{k=1}^{m} \|\mathcal{I}_k \mu_1(\zeta_k^-) - \mathcal{I}_k \bar{\mu}_1(\zeta_k^-)\| \\
& \leq \mathbb{A}_1 (q-1)\rho_1^{*(q-2)}(m+1) \Bigg[[\mathbb{B}_1 (m+1)\mathcal{K} \|\mu_1 \\
& - \bar{\mu}_1\| + m\mathcal{O}\|\mu_1 - \bar{\mu}_1\|] + \frac{\mathbb{B}_2 (m+1)}{\lambda_1} \mathcal{L} \\
& \times \|\mu_1 - \bar{\mu}_1\| \Bigg] + \frac{\mathbb{A}_1 (q-1)\rho_1^{*(q-2)}(m+1)}{\Gamma(\sigma_1)} \\
& \times \Bigg[\mathbb{B}_1 (m+1)\mathcal{K}\|\mu_1 - \bar{\mu}_1\| + m\mathcal{O} \\
& \times \|\mu_1 - \bar{\mu}_1\| + \frac{\mathbb{B}_1 (m+1)}{\lambda_1} \mathcal{L} \|\mu_1 - \bar{\mu}_1\| \Bigg] \\
& + m\mathcal{O}\|\mu_1 - \bar{\mu}_1\| \\
& \leq \Bigg[\Bigg[\mathbb{A}_1 (q-1)\rho_1^{*(q-2)}(m+1) \\
& + \frac{\mathbb{A}_1 (q-1)\rho_1^{*(q-2)}(m+1)}{\Gamma(\sigma_1)} \Bigg] \Bigg[\mathbb{B}_1 (m+1)\mathcal{K} \\
& \times + \frac{\mathbb{B}_1 (m+1)}{\lambda_1} \mathcal{L} + m\mathcal{O} \Bigg] + m\mathcal{O} \Bigg] \\
& \times \|\mu_1 - \bar{\mu}_1\|.
\end{aligned} \tag{3.12}
$$

Consequently, with the aid of Case 1 to Case 2 and condition (3.10), we obtain $\mathcal{F}$ is a contraction. With the aid of Banach's fixed-point theorem, it implies that the fixed point is unique, which further implies that the impulsive system involving ABC-derivative and Caputo–Fabrizio derivative has a unique solution. □

4. HYERS–ULAM STABILITY OF SOLUTION

In this part, we provide adequate and necessary conditions for HU-stability for the solution of impulsive system FDEs.

Theorem 6. *With the aid of the conditions* $(\mathbb{H}_1)$–$(\mathbb{H}_3)$, *the impulsive system with ABC-derivative and Caputo–Fabrizio derivative* (1.1) *is Hyers–Ulam stable.*

Proof. With the aid of Theorem 3, we have

$$\mu_1(\zeta)-\tilde{\mu}_1(\zeta) = \begin{cases} \psi_0+\mathbb{A}_1\phi_q(\mathbb{G}(\mu_1(\zeta)))+\mathbb{A}_2\int_0^{\zeta}(\zeta-\tau)^{\sigma_1-1}(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau, & \text{for } \zeta\in[0,\zeta_1],\\ \psi_0+\mathbb{A}_1[\mathbb{G}(\mu_1(\zeta_1))+\mathbb{G}(\mu_1(\zeta))]+\mathbb{A}_2\left[\int_0^{\zeta_1}(\zeta_1-\tau)^{\sigma_1-1}(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau\right.\\ \qquad\left.+\int_{\zeta_1}^{\zeta}(\zeta-\tau)^{\sigma_1-1}(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau\right]+\mathcal{I}_1\mu_1(\zeta_1^-), & \text{for } \zeta\in[\zeta_1,\zeta_2],\\ \vdots\\ \psi_0+\mathbb{A}_1\sum_{k=1}^{m+1}\mathbb{G}(\mu_1(\zeta_k))+\mathbb{A}_2\left[\sum_{k=1}^{m}\int_{\zeta_{k-1}}^{\zeta_k}(\zeta_k-\tau)^{\sigma_1-1}(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau\right.\\ \qquad\left.+\int_{\zeta_m}^{\zeta}(\zeta-\tau)^{\sigma_1-1}(\phi_q(\mathbb{G}(\mu_1(\tau))))d\tau\right]+\sum_{k=1}^{m}\mathcal{I}_k\mu_1(\zeta_k^-), & \text{for } \zeta\in[\zeta_m,\mathcal{T}]. \end{cases} \tag{4.1}$$

Then for $\zeta\in[0,\zeta_1]$, we have

$$\begin{aligned}
&\|\mu_1(\zeta)-\tilde{\mu}_1(\zeta)\|\\
&\quad\le \mathbb{A}_1\|\phi_q(\mathbb{G}(\mu_1(\zeta)))-\phi_q(\mathbb{G}(\tilde{\mu}_1(\zeta)))\|\\
&\qquad+\mathbb{A}_2\int_0^{\zeta}(\zeta-\tau)^{\sigma_1-1}\|(\phi_q(\mathbb{G}(\mu_1(\tau))))\\
&\qquad-(\phi_q(\mathbb{G}(\tilde{\mu}_1(\tau))))\|d\tau\\
&\quad\le \mathbb{A}_1(q-1)\rho_1^{*(q-2)}\Bigg[\mathbb{B}_1\|\mathcal{Y}_1^*(\zeta,\mu_1(\zeta))\\
&\qquad-\mathcal{Y}_1^*(\zeta,\tilde{\mu}_1(\zeta))\|+\mathbb{B}_2\\
&\qquad\times\left\|\int_0^{\zeta}(\zeta-s)^{\lambda_1-1}\right\|\|\mathcal{Y}_1^*(s,\mu_1(s))\\
&\qquad-\mathcal{Y}_1^*(s,\tilde{\mu}_1(s))\|ds\Bigg]+\frac{\mathbb{A}_2(q-1)\rho_1^{*(q-2)}}{\Gamma(\sigma_1)}\\
&\qquad\times\Bigg[\mathbb{B}_1\|\mathcal{Y}_1^*(\tau,\mu_1(\tau))-\mathcal{Y}_1^*(\tau,\tilde{\mu}_1(\tau))\|\\
&\qquad+\mathbb{B}_2\left\|\int_0^{\zeta}(\zeta-s)^{\lambda_1-1}\right\|\|\mathcal{Y}_1^*(s,\mu_1(s))\\
&\qquad-\mathcal{Y}_1^*(s,\tilde{\mu}_1(s))\|ds\Bigg]\\
&\quad\le\left[\left[\mathbb{A}_1(q-1)\rho_1^{*(q-2)}+\frac{\mathbb{A}_2(q-1)\rho_1^{*(q-2)}}{\Gamma(\sigma_1)}\right]\right.\\
&\qquad\left.\times\left(\mathbb{B}_1\mathcal{K}+\frac{\mathbb{B}_2}{\lambda_1}\mathcal{L}\right)\right](\|\mu_1-\tilde{\mu}_1\|).
\end{aligned}\tag{4.2}$$

Now for $\zeta\in[\zeta_k,\zeta_{k+1}]$, where $k=1,2,\ldots,m$, we have

$$\begin{aligned}
&\|\mu_1(\zeta)-\tilde{\mu}_1(\zeta)\|\\
&\quad\le\mathbb{A}_1\sum_{k=1}^{m+1}\|\phi_q\mathbb{G}(\mu_1(\zeta_k))-\phi_q\mathbb{G}(\tilde{\mu}_1(\zeta_k))\|\\
&\qquad+\mathbb{A}_2\left[\sum_{k=1}^{m}\int_{\zeta_{k-1}}^{\zeta_k}(\zeta_k-\tau)^{\sigma_1-1}\|\phi_q(\mathbb{G}(\mu_1(\tau)))\right.\\
&\qquad-\phi_q(\mathbb{G}(\tilde{\mu}_1(\tau)))\|d\tau\\
&\qquad+\int_{\zeta_m}^{\zeta}(\zeta-\tau)^{\sigma_1-1}\|\phi_q(\mathbb{G}(\mu_1(\tau)))\\
&\qquad\left.-\phi_q(\mathbb{G}(\tilde{\mu}_1(\tau)))\|d\tau\right]\\
&\qquad+\sum_{k=1}^{m}\|\mathcal{I}_k\mu_1(\zeta_k^-)-\mathcal{I}_k\tilde{\mu}_1(\zeta_k^-)\|\\
&\quad\le\mathbb{A}_1(q-1)\rho_1^{*(q-2)}\\
&\qquad\times\sum_{k=1}^{m+1}\Bigg[\mathbb{B}_1\sum_{k=1}^{m+1}\|\mathcal{Y}_1^*(\zeta_k,\mu_1(\zeta_k))\\
&\qquad-\mathcal{Y}_1^*(\zeta_k,\tilde{\mu}_1(\zeta_k))\|
\end{aligned}$$

$$+\sum_{k=1}^{m}\|\mathcal{I}_k\mu_1(\zeta_k^-)-\mathcal{I}_k\tilde{\mu}_1(\zeta_k^-)\|$$

$$+\mathbb{B}_2\sum_{k=1}^{m}\int_{\zeta_{k-1}}^{\zeta_k}(\zeta_k-s)^{\lambda_1-1}\|\mathcal{Y}_1^*(s,\mu_1(s))$$

$$-\mathcal{Y}_1^*(s,\tilde{\mu}_1(s))\|ds\int_{\zeta_m}^{\zeta}(\zeta-s)^{\lambda_1-1}$$

$$\times\|\mathcal{Y}_1^*(s,\mu_1(s))-\mathcal{Y}_1^*(s,\tilde{\mu}_1(s))\|ds\Bigg]$$

$$+\mathbb{A}_1(q-1)\rho_1^{*(q-2)}\Bigg[\sum_{k=1}^{m}\int_{\zeta_{k+1}}^{\zeta_k}(\zeta_k-\tau)^{\sigma_1-1}$$

$$\times\Bigg[\mathbb{B}_1\sum_{k=1}^{m+1}\|\mathcal{Y}_1^*(\tau_k,\mu_1(\tau_k))-\mathcal{Y}_1^*(\tau_k,\tilde{\mu}_1(\tau_k))\|$$

$$+\sum_{k=1}^{m}\|\mathcal{I}_k\mu_1(\tau_k^-)-\mathcal{I}_k\tilde{\mu}_1(\tau_k^-)\|$$

$$+\mathbb{B}_2\sum_{k=1}^{m}\int_{\tau_{k-1}}^{\tau_k}(\tau_k-s)^{\lambda_1-1}\|\mathcal{Y}_1^*(s,\mu_1(s))$$

$$-\mathcal{Y}_1^*(s,\tilde{\mu}_1(s))\|ds+\mathbb{B}_2\int_{\tau_m}^{\tau}(\tau-s)^{\lambda_1-1}$$

$$\times\|\mathcal{Y}_1^*(s,\mu_1(s))-\mathcal{Y}_1^*(s,\tilde{\mu}_1(s))\|ds\Bigg]d\tau$$

$$+\int_{\zeta_m}^{\zeta}(\zeta-\tau)^{\sigma_1-1}\Bigg[\mathbb{B}_1\sum_{k=1}^{m+1}\|\mathcal{Y}_1^*(\tau_k,\mu_1(\tau_k))$$

$$-\mathcal{Y}_1^*(\tau_k,\tilde{\mu}_1(\tau_k))\|(\zeta-s)^{\lambda_1-1}-\mathcal{I}_k\tilde{\mu}_1(\tau_k^-)\|$$

$$+\mathbb{B}_2\sum_{k=1}^{m}\int_{\tau_{k-1}}^{\tau_k}(\tau_k-s)^{\lambda_1-1}\|\mathcal{Y}_1^*(s,\mu_1(s))$$

$$-\mathcal{Y}_1^*(s,\tilde{\mu}_1(s))\|ds(\zeta-s)^{\lambda_1-1}\|\mathcal{Y}_1^*(s,\mu_1(s))$$

$$-\mathcal{Y}_1^*(s,\tilde{\mu}_1(s))\|ds\Bigg]d\tau\Bigg]+\sum_{k=1}^{m}\|\mathcal{I}_k\mu_1(\zeta_k^-)$$

$$-\mathcal{I}_k\tilde{\mu}_1(\zeta_k^-)\|$$

$$\leq\Bigg[\Bigg[\mathbb{A}_1(m+1)(q-1)\rho_1^{*(q-2)}$$

$$+\frac{\mathbb{A}_2(m+1)(q-1)\rho_1^{*(q-2)}}{\Gamma(\sigma_1)}\Bigg]\Bigg[\mathbb{B}_1(m+1)\mathcal{K}$$

$$+\frac{\mathbb{B}_2(m+1)}{\lambda_1}\mathcal{L}+(m+1)\mathcal{O}\Bigg]+m\mathcal{O}\Bigg]$$

$$\times\|\mu_1-\tilde{\mu}_1\|. \tag{4.3}$$

□

As a result, with the aid of Eqs. (4.2) and (4.3), the impulsive system involving ABC-derivative and Caputo–Fabrizio derivative (1.1) is Hyers–Ulam stable.

5. APPLICATION

The following example illustrates the application of the theorems stated in Secs. 3 and 4.

Example 1. Suppose that $\mathcal{Y}_1^*(\zeta,\mu_1(\zeta))=\frac{|\mu_1(\zeta)|}{100+|\mu_1(\zeta)|},\zeta\in[0,1]$. Consider the following impulsive FDEs as an example of (3.1):

$$\begin{cases} {}_0^{ABC}\mathcal{D}^{0.4}(\phi_5({}_0^{CF}\mathcal{D}^{0.4}(\mu_1(\zeta)))=\mathcal{Y}_1^*(\zeta,\mu_1(\zeta)), \\ \qquad\qquad\qquad \zeta\in\aleph=[0,\mathcal{T}], \\ \Delta[\phi_5({}_0^{CF}\mathcal{D}^{0.4}(\mu_1(\zeta_k)))] \\ \quad =\mu_1(\zeta_k^+)-\mathcal{I}_k(\zeta_k^-)=\mu_1(\zeta_k^-), \\ \Delta\mu_1(\zeta_k)=\mu_1(\zeta_k^+)-\mathcal{I}_k(\zeta_k^-)=\mu_1(\zeta_k^-), \\ [\phi_p({}_0^{CF}\mathcal{D}^{0.4}(\mu_1(0)))]=\psi_0,\quad \mu(0)=\mu_0. \end{cases} \tag{5.1}$$

For $p=5,\rho_1^*=0.5,\sigma_1=0.4,\lambda_1=0.4,m=30,k=1,2,\ldots,30$ which implies $\mathcal{L}=\mathcal{K}=\mathcal{O}=0.001$. The conditions $(\mathbb{H}_1)-(\mathbb{H}_3)$ are satisfied and $\varpi^*<1$.

Case 1. For $\zeta\in[0,\zeta_1]$ where $\zeta_1\in[0,1]$

$$\varpi^*=\Bigg[\Bigg[\mathbb{A}_1(q-1)\rho_1^{*(q-2)}+\frac{\mathbb{A}_2(q-1)\rho_1^{*(q-2)}}{\Gamma(\sigma_1)}\Bigg]$$

$$\times\left(\mathbb{B}_1\mathcal{K}+\frac{\mathbb{B}_2\mathcal{L}}{\lambda_1}\right)\Bigg](\|\mu_1-\bar{\mu}_1\|)<1,$$

$$\varpi^*=\Bigg[\Bigg[\frac{2(1-\sigma_1)(q-1)\rho_1^{*(q-2)}}{2\mathbb{M}(\sigma_1)-\sigma_1\mathbb{M}(\sigma_1)}$$

$$+\frac{2\sigma_1(q-1)\rho_1^{*(q-2)}}{2\mathbb{M}(\sigma_1)-\sigma_1\mathbb{M}(\sigma_1)\Gamma(\sigma_1)}\Bigg]$$

$$\times\left(\frac{(1-\lambda_1)\mathcal{K}}{\mathbb{B}(\lambda_1)}+\frac{\lambda_1\mathcal{L}}{\mathbb{B}(\lambda_1)\Gamma(\lambda_1+1)}\right)\Bigg]$$

$$=0.0066602<1. \tag{5.2}$$

Case 2. For $\zeta \in [\zeta_k, \zeta_{k+1}]$

$$\varpi^* = \Bigg[\Bigg[\Bigg[\mathbb{A}_1(m+1)(q-1)\rho_1^{*(q-2)} + \frac{\mathbb{A}_2(m+1)(q-1)\rho_1^{*(q-2)}}{\Gamma(\sigma_1)}\Bigg]\Bigg[\mathbb{B}_1(m+1)\mathcal{K} + \frac{\mathbb{B}_2(m+1)}{\lambda_1}\mathcal{L} + m\mathcal{O}\Bigg] + m\mathcal{O}\Bigg] < 1,$$

$$\varpi^* = \Bigg[\Bigg[\frac{2(1-\sigma_1)(m+1)(q-1)\rho_1^{*(q-2)}}{2\mathbb{M}(\sigma_1) - \sigma_1\mathbb{M}(\sigma_1)} + \frac{2\sigma_1(m+1)(q-1)\rho_1^{*(q-2)}}{2\mathbb{M}(\sigma_1) - \sigma_1\mathbb{M}(\sigma_1)\Gamma(\sigma_1)}\Bigg] \times \Bigg[\frac{(1-\lambda_1)(m+1)}{\mathbb{B}(\lambda_1)}\mathcal{K} + \frac{\lambda_1(m+1)}{\mathbb{B}(\lambda_1)\Gamma(\lambda_1+1)}\mathcal{L} + m\mathcal{O}\Bigg] + m\mathcal{O}\Bigg] = 0.7392 < 1. \tag{5.3}$$

By Theory 3 and Eqs. (5.2), (5.3), we deduce that the impulsive system (1.1) has a unique positive solution. Thus, similarly, the impulsive system fractional differential involving ABC-derivative and Caputo–Fabrizio derivative (4.3) is Hyers–Ulam stable.

Here, we use the new numerical scheme to show the numerical simulation of the impulsive system fractional differential with ABC-derivative and Caputo–Fabrizio derivative on (0.1].

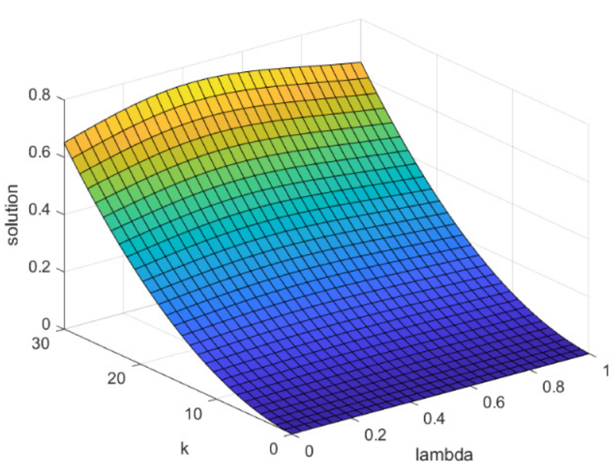
We illustrated our results for an impulsive system by using MATLAB Ver. 9.10.0.1602886 (R2021a)[57] at different fractional orders. We divided the proof into the following cases:

Case 1. We designated one value to ABC-derivative where $\sigma_1 = 0.4$, while the Caputo–Fabrizio derivative λ_1 is calculated at different fractional orders $\lambda_1 = 0, \ldots, 1$. In Fig. 1a, $\varpi^*_{1i}, i = 1, \ldots, 30$ are the solutions of our proposed system. The solutions are investigated at various values k, where $k = 1, \ldots, 30$.

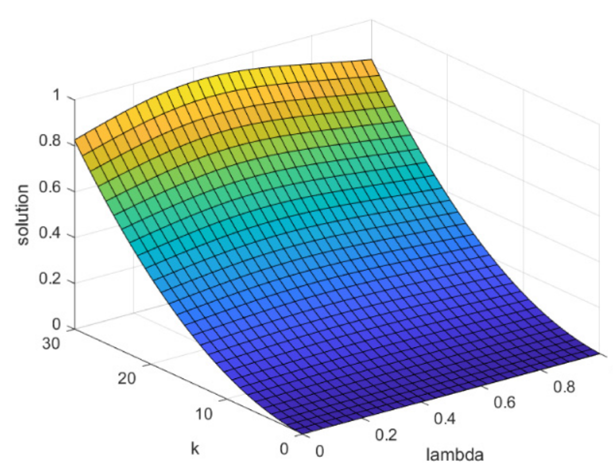
We designated one value to ABC-derivative where $\sigma_1 = 1$, while the Caputo–Fabrizio derivative λ_1 is calculated at different fractional orders $\lambda_1 = 0, \ldots, 1$. In Fig. 1b, $\varpi^*_{1i}, i = 1, \ldots, 30$ are the solutions of our proposed system. The solutions are investigated at various values k, where $k = 1, \ldots, 30$.

Case 2. We designated one value to Caputo–Fabrizio derivative where $\lambda_1 = 0.4$, while the ABC-derivative σ_1 is calculated at different fractional orders $\sigma_1 = 0, \ldots, 1$. In Fig. 2a, $\varpi^*_{2i}, i = 1, \ldots, 30$ are the solutions of our proposed system. The solutions are investigated at various values k, where $k = 1, \ldots, 30$.

We designated one value to Caputo–Fabrizio derivative where $\lambda_1 = 1$, while the ABC-derivative σ_1 is calculated at different fractional orders $\sigma_1 = 0, \ldots, 1$. In Fig. 2b, $\varpi^*_{2i}, i = 1, \ldots, 30$ are the solutions of our proposed system. The solutions

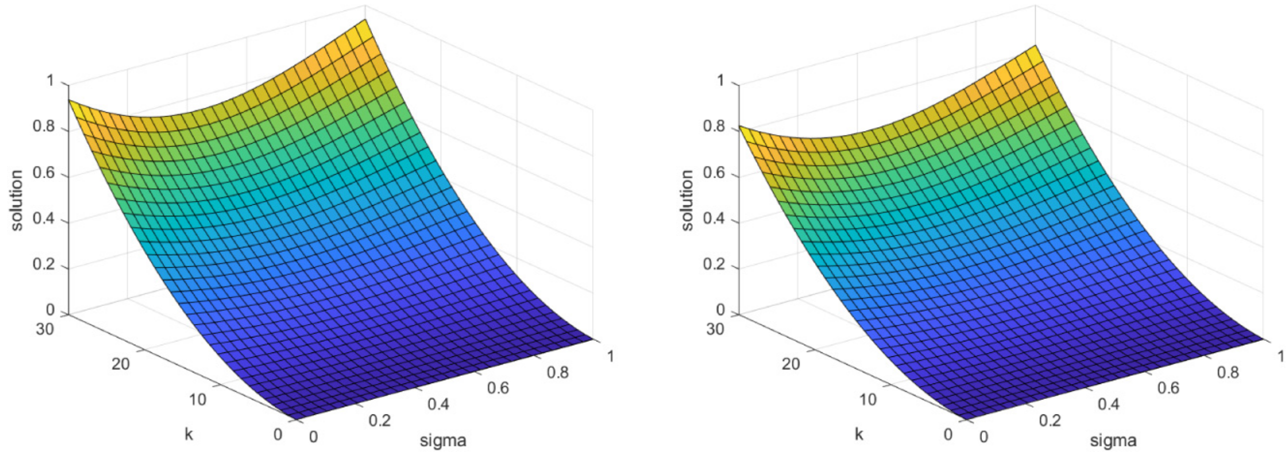


(a)



(b)

Fig. 1 The approximate solutions of the proposed impulsive system: graph **(a)** represents ϖ^* at $\sigma_1 = 0.4$ and $\lambda_1 = 0, \ldots, 1$; graph **(b)** represents ϖ^* at $\sigma_1 = 1$ and $\lambda_1 = 0, \ldots, 1$.



(a) **(b)**

Fig. 2 The approximate solutions of the proposed impulsive system: graph **(a)** represents ϖ^* at $\lambda_1 = 0.4$ and $\sigma_1 = 0, \ldots, 1$; graph **(b)** represents ϖ^* at $\lambda_1 = 1$ and $\sigma_1 = 0, \ldots, 1$.

are investigated at various values k, where $k = 1, \ldots, 30$.

We conclude that ϖ^* represents a better approximation to our exact solution for the proposed impulsive system.

6. CONCLUSIONS

We focused on establishing existence and uniqueness in this literature by using Leray–Schauder's theory. We were able to provide necessary conditions for the existence of a solution for an impulsive system FDEs involving ABC-derivative and Caputo–Fabrizio derivatives. For these purposes, we converted our problem into the form of integral equations. Then, we investigated the existence of solutions with the aid of Leray–Schauder's theory. We verified the uniqueness by using the Banach principle. Also, we investigated the HU-stability of the solution. To demonstrate the results, we included an example to support our results. We recommend that future work investigates the problem of the Controllability of semilinear impulsive FDEs by Caputo–Fabrizio derivative.

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