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Numerical Simulation of the Oil Production Performance of Different Well Patterns with Water Injection

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Abstract: Numerical reservoir simulation, which includes the construction and operation of a model that performs similarly to a real-world reservoir, is an effective method for exploring complex reservoir issues. Due to the complexity of constructing reservoir environments for experiments, numerical simulation is a vital method for studying flow behavior under reservoir conditions. In this study, a black-oil modeling simulator was used to construct, simulate, and evaluate a conceptual hydrocarbon reservoir model. The model evolved by drilling two production wells and one injection well in two cases. The first case consisted of two horizontal production wells and one injection well, while the second consisted of two vertical production wells and an injection well. In total, 25 simulation runs were performed, and the results showed that horizontal wells perform better than vertical wells in terms of productivity, with a field oil production total of 1,930,000 m^3. This is significantly higher than vertical wells, which have a field oil production total of 1,890,000 m^3 after 1840 days. The field recovery factor for horizontal wells was 41% and for vertical wells it was 39%, both of which were less than 50%. This indicates that the reservoir's sweeping efficiency was minimal. To enhance sweeping efficiency, the water injection rate and number of injection wells should be increased, as well as well patterns and locations remodeled. It was also shown that as reservoir thickness increased, horizontal and vertical well productivity increased. In order to boost horizontal well productivity and increase field oil recovery above 50%, the horizontal well length should be increased to take up a wider area of the reservoir portion. On the other hand, well length may have no impact on vertical well production efficiency.

Keywords: horizontal and vertical wells; productivity; production performance; reservoir model; numerical simulation

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1. Introduction

During the initial phases of well planning and development for petroleum production, engineers must decide whether to start producing hydrocarbons using a vertical or horizontal well for a particular reservoir, with unique characteristics that either type of drilling and production technique could favor [1]. A reservoir with a reasonable height, thickness, and high surface area will require the use of a particular well type to effectively drain it [2]. When confronted with this problem, production engineers and reservoir engineers must decide whether to produce hydrocarbons from various vertical wells or just one horizontal well on the site to cover the reservoir area sufficiently for proper drainage [3].

Many considerations influence the selection of production technique options, which include reservoir properties such as shape, height, length, and dip angle; economics; projected financial returns on investment depending on the productivity of the drilled wells; and so on [4,5]. Oil and gas production from wells is commonly expressed in terms of well productivity, which is influenced by a variety of aspects including reservoir structure, completion category, petrophysical and fluid characteristics, formation damage, and so on [6]. A hydrocarbon well's productivity index is often determined by the pressure losses between the reservoir edge and the well bore [7–10]. Reservoir drainage area, pay zone thickness, anisotropy, well length, flow rate, and well completion techniques are all things that influence the productivity index [3,11,12].

A number of researchers have conducted studies trying to compare the production performance of horizontal and vertical wells. Dankwa et al. (2018) compared the economic analysis and performance of horizontal and vertical wells and reported that horizontal wells achieve a higher level of productivity than vertical wells, but they are more costly to drill, accomplish, and produce [3]. Jamiolahmady et al. (2007) published a paper that compared the productivity of vertical, slanted, and horizontal wells in layered gas reservoirs. They carried out a series of evaluations on a single-well model using a compositional reservoir simulator. According to their findings, horizontal wells have higher productivities in homogeneous systems [13]. Soleimani et al. (2018) released a study on the responsiveness of horizontal well production efficiency in tight gas formations to reservoir properties. Sherrard (1995) also performed studies in the North Slope of Alaska to forecast and evaluate the performance of horizontal wells. This research discovered that horizontal well performance and assessment are inherently more difficult than vertical well performance. However, their study also found that horizontal wells do not always enhance coning performance, despite lower drawdown [14].

The objective of this study was to use black-oil modelling to build a conceptual reservoir model that helped to compare vertical and horizontal wells in terms of oil production performance [15,16]. Black oil is a fluid model in which water is explicitly modeled together with two hydrocarbon components, one (pseudo) oil phase and one (pseudo) gas phase [17,18]. The basic equations for the black-oil model consist of (1)–(3) [19,20]. The equations below describe fluid flow in a petroleum reservoir, constituting the mathematical framework for a black-oil reservoir simulator [19,21].

$$\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) + \frac{\partial}{\partial t}\left(\frac{\phi R_V S_g}{B_g}\right) + \nabla\cdot\left(\frac{1}{B_o}\vec{u}_o\right) + \nabla\cdot\left(\frac{R_V}{B_g}\vec{u}_g\right) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}\left(\frac{\phi S_g}{B_g}\right) + \frac{\partial}{\partial t}\left(\frac{\phi R_S S_o}{B_o}\right) + \nabla\cdot\left(\frac{1}{B_g}\vec{u}_g\right) + \nabla\cdot\left(\frac{R_S}{B_o}\vec{u}_o\right) = 0 \tag{2}$$

$$\frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right) + \nabla\cdot\left(\frac{1}{B_w}\vec{u}_w\right) = 0 \tag{3}$$

2. Simulation

2.1. Methodologies

In this section, a step-by-step approach is given to set up a 3D reservoir simulation model. Although this was a conceptual model, it was complex enough to show all the basic ideas involved in reservoir simulation. Therefore, the model was heterogeneous, with different permeabilities in different layers. The black-oil model, with the help of the Schlumberger ECLIPSE 100 simulator, was used to build, simulate, and analyze the required hydrocarbon reservoir conceptual model. The ECLIPSE 100 was chosen for this study because it solves the black-oil equations (a fluid model) on corner-point grids.

The conceptual reservoir was a layered oil reservoir with water injection. The reservoir was divided into 10 layers with varying permeabilities (heterogeneous) of equal thickness. The layers were based on a 30 × 30 × 10 grid, which made a total of 9000 cells. The dimensions of the grid blocks were 20 m, 20 m, and 8 m in the x, y, and z directions, respectively. The permeability used a heterogeneous model, TOPS: 2500 m TVD, oil–water contact: 2600 m, gas–oil contact: 2300 m; the density of the oil, water, and gas was 850 kg/m^3, 1000 kg/m^3, and 0.756 kg g/m^3 respectively. All the characteristics of the model and other relevant data used to construct and simulate the model are given in Tables 1–3, using metric units throughout.

Table 1. Reservoir characteristics of the model.

Parameters	Layers 1–5	Layers 6–10
Blocks	9000	9000
Reservoir top depth	2500	2500
Porosity	0.3	0.3
Permeability X	150	500
Permeability Y	600	1000
Permeability Z	20	60

Table 2. Water–oil and gas–oil relative permeabilities.

Water–Oil Relative Permeability				Gas–Oil Relative Permeability			
S_w	K_{rw}	K_{ro}	P_c	S_g	K_{rg}	K_{ro}	P_c
0.38	0	1	0.0011	0	0	1	0
0.44092	0.01984	0.69851	0.0006	0.03639	0.0145	0.82626	0
0.48205	0.04111	0.4844	0.00041	0.07278	0.029	0.65253	0
0.53806	0.08559	0.18307	0.00027	0.10917	0.0435	0.47879	0
0.5709	0.1227	0.0747	0.00024	0.14556	0.05942	0.32133	0
0.588	0.1512	0.0493	0.00022	0.18194	0.07844	0.20468	0
0.6086	0.1857	0.0347	0.00021	0.21833	0.09907	0.11951	0
0.6376	0.2367	0.0216	0.00019	0.25472	0.1299	0.08211	0
0.6698	0.2994	0.0131	0.00018	0.29111	0.1612	0.05631	0
0.7008	0.3681	0.0083	0.00016	0.3275	0.19522	0.03839	0
0.7553	0.5136	0.0038	0.00015	0.36389	0.2325	0.02522	0
0.8234	0.7503	0	0.00014	0.40028	0.27369	0.01593	0
1	1	0	0	0.43667	0.31991	0.00916	0
				0.47306	0.36928	0.00447	0
				0.50944	0.4228	0.00128	0
				0.54583	0.47867	0	0
				0.65	1	0	0

Table 3. PVT properties of oil and gas.

PVT Properties of Oil				PVT Properties of Gas		
R_s	P_{bub}	B_o	μ_o	P	B_g	μ_g
0.275	400	1.13	1.17	400	5.9	0.013
0.938	2000	1.162	1.11	800	2.95	0.0135
1.5	3600	1.243	0.95	1200	1.96	0.014
1.5	4000	1.238	0.95	1600	1.47	0.0145
1.5	4400	1.233	0.95	2000	1.18	0.015
1.5	4800	1.228	0.95	2400	0.98	0.0155
1.5	5200	1.223	0.95	2800	0.84	0.016
1.5	5600	1.218	0.95	3200	0.74	0.0165
1.72	4400	1.254	0.94	3600	0.65	0.017
1.72	4800	1.266	0.92	4000	0.59	0.0175
1.72	5200	1.26	0.92	4400	0.54	0.018
1.72	5600	1.25	0.92	4800	0.49	0.0185
				5200	0.45	0.019
				5600	0.42	0.0195

The ECLIPSE data files with relevant information were specified and created and then imported into the ECLIPSE 100 Schlumberger black-oil model. Two models were created for two different cases: (1) a field model with horizontal production wells and (2) a field model with vertical production wells. The vertical wells were drilled first, followed by the horizontal wells. The model was run for both vertical and horizontal wells to observe the results. These results included reservoir shape and size, horizontal and vertical drilling, and other parameters, as shown in Figures 1–3 below.

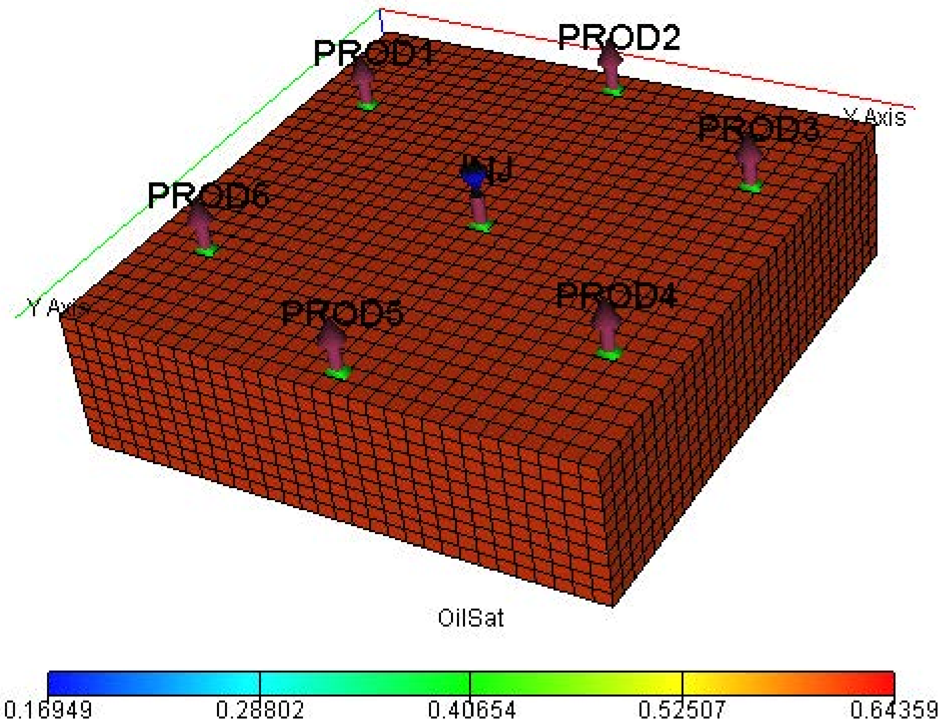

Figure 1. Shape of the reservoir.

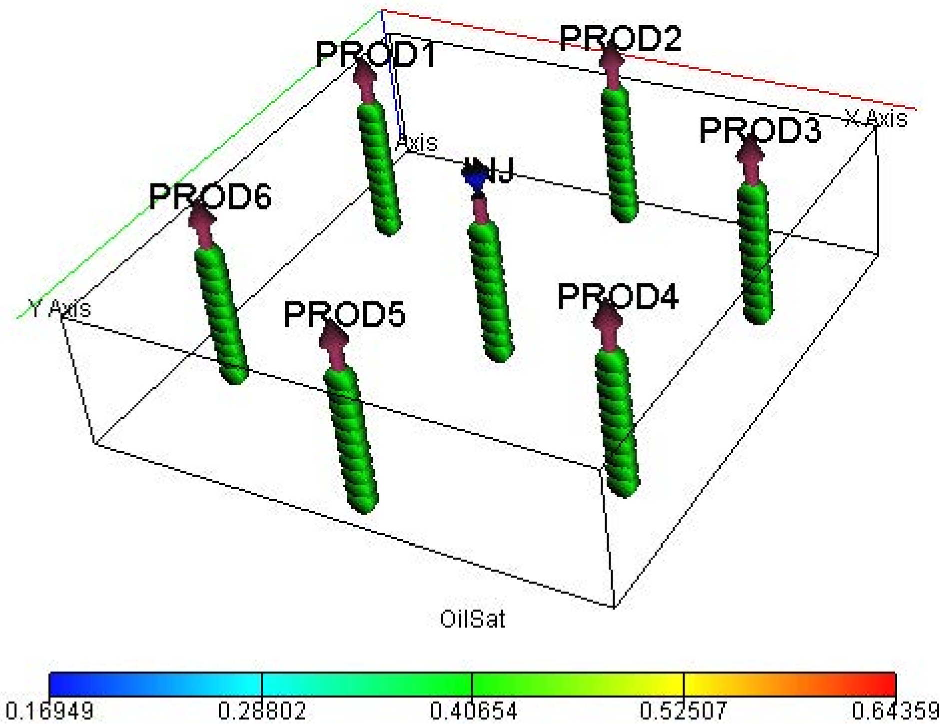

Figure 2. Vertical wells.

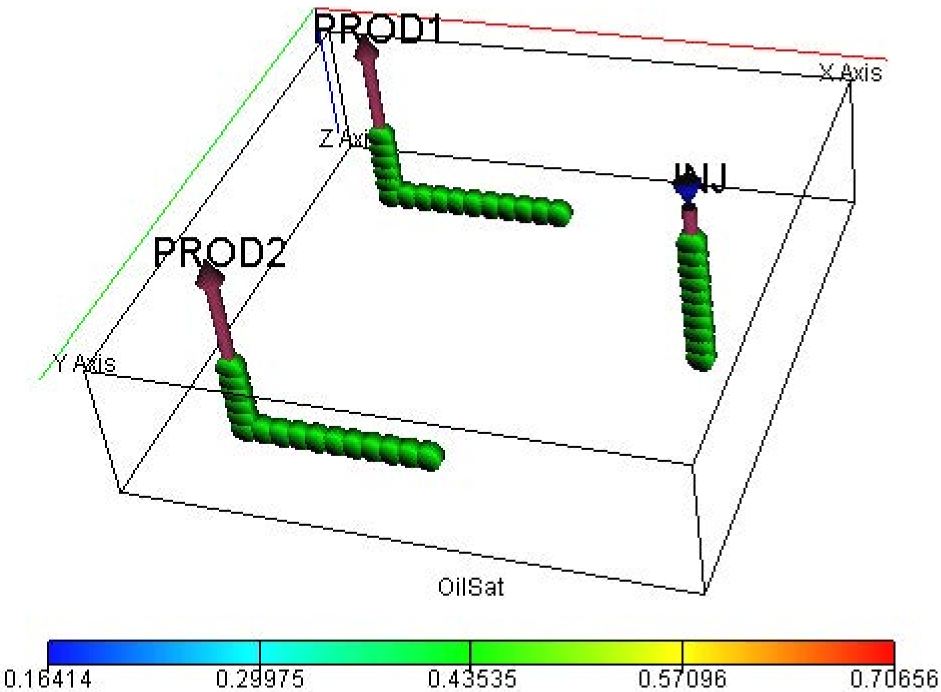

Figure 3. Horizontal wells.

2.2. Simulation Procedures

Two production wells (PR1 and PR2) belonging to group OP and one injector well (INJ1) belonging to group WI were drilled for both cases presented. The inside diameter of the wells was 0.2 m. Oil was produced at a rate of 3000 m^3/day of liquid for the production wells and 3352 m^3/day for the injection well. Time for the simulation was set to a value of 92 steps, where each step was assumed to be 20 days. The simulation started on 27 June 2021.

2.2.1. Case 1: Horizontal Production Wells and Injection Well

In this case, two horizontal production wells were drilled at block no. (5, 5) to (6–15, 6) and (5, 25) to (6–15, 26), respectively, and one injection well was drilled in block no. (26, 15) (Figure 4).

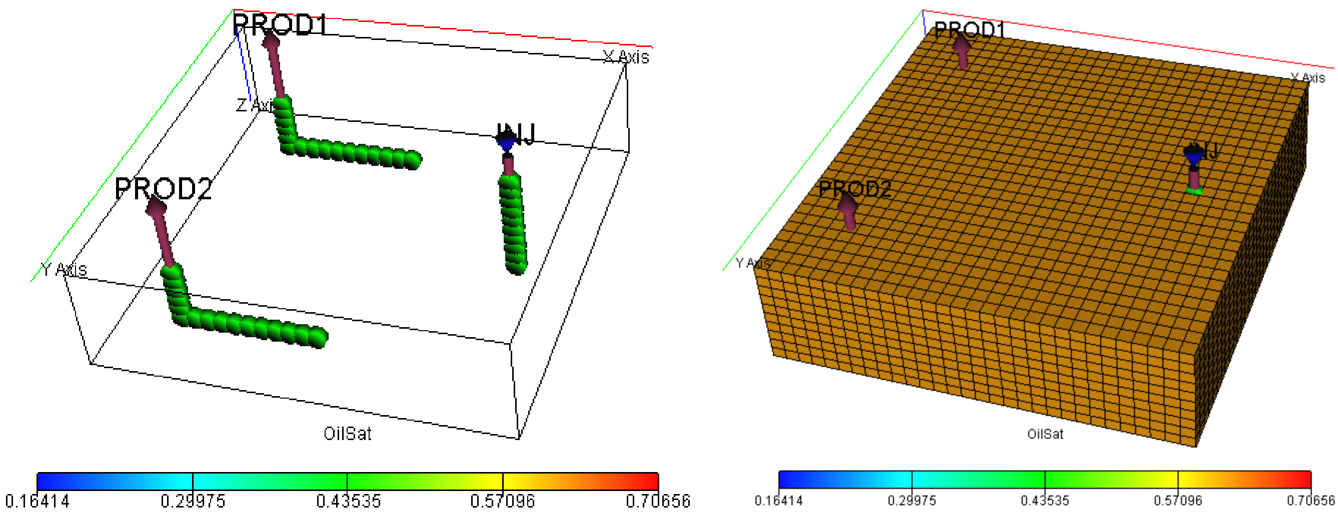

Figure 4. Map view of the location and position of two horizontal production wells and one vertical injection well.

2.2.2. Case 2: Vertical Production Wells and Injection Well

In this case, two vertical production wells and one injector well were drilled in blocks no. (5, 5), (5, 25), and (26, 15), respectively (Figure 5 below).

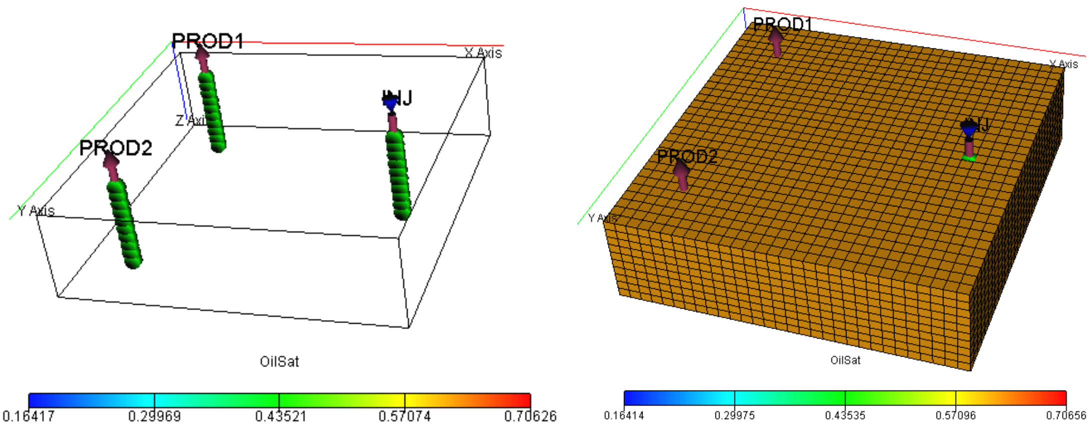

Figure 5. Map view of the location and position of two vertical production wells and one injection well.

2.2.3. Model Permeability Distribution

According to the obtained results, this conceptual model was heterogeneous, which means it had different permeabilities in different layers, as shown in Figure 6.

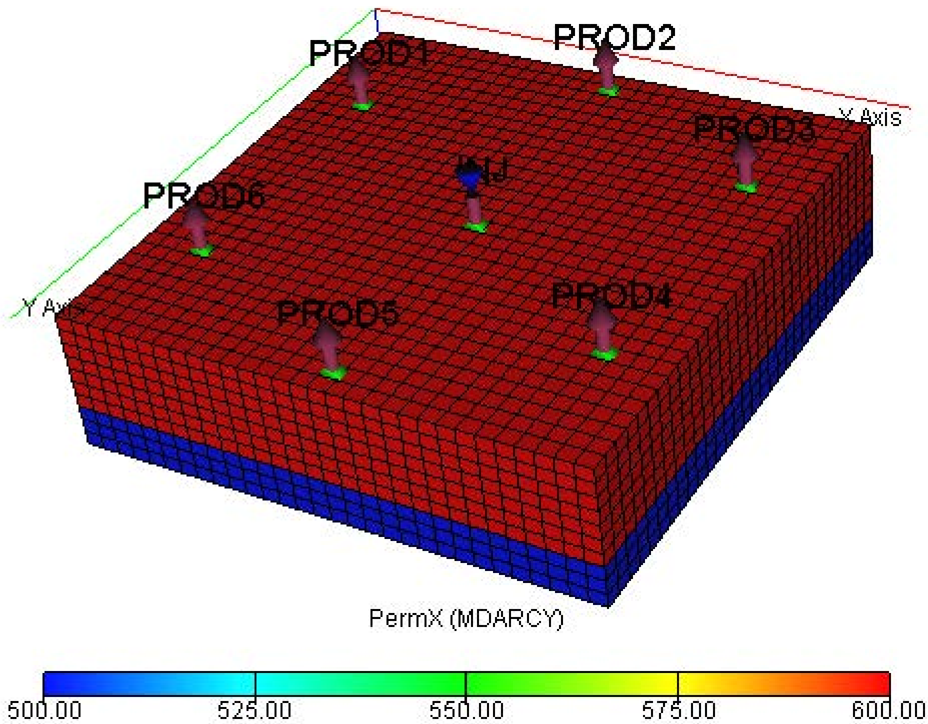

Figure 6. Permeability distribution.

3. Analysis of the Results

This section represents all results and discussions after conduction of the numerical reservoir simulation using black-oil modelling. All key results obtained from the numerical simulation were plotted, and their explanations are provided below.

3.1. Oil Production from Horizontal Wells

3.1.1. Field Oil Production

Oil wells usually produce at their maximum rate at the start of their lives; the production rate eventually declines to a point at which they no longer produce profitable amounts. The shape of the decline curve can be explained by the oil reservoir and the reservoir drive mechanism. The simulated field production rates versus time are shown in Figure 7. It can be seen that field oil production decreased with time from a maximum of 6000 m^3/day to 196 m^3/day at 1840 days. As can be seen in Figure 8, the rate of FOPT increased dramatically over time, rising to the maximum amount of 1,937,523.9 m^3 at 1840 days (5 years). After two horizontal production wells and a vertical injector well were drilled, oil recovery was found to be 41% for 5 years, as shown in Figure 9. This indicates that sweep efficiency was poor and a considerable amount of oil was unrecovered.

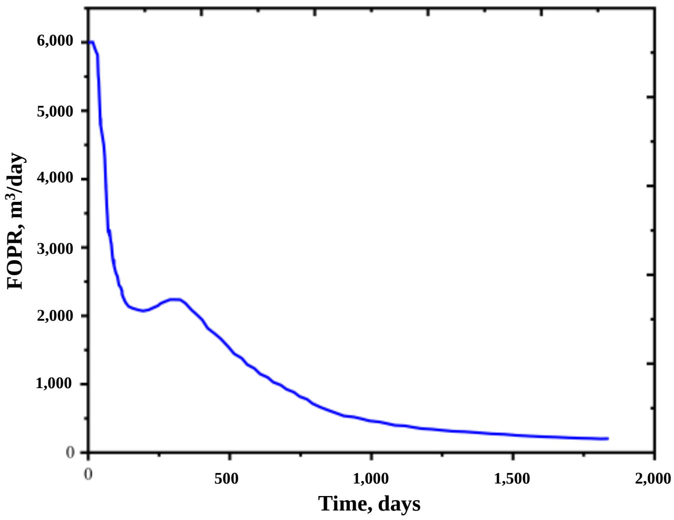

Figure 7. Field oil production rate for horizontal wells.

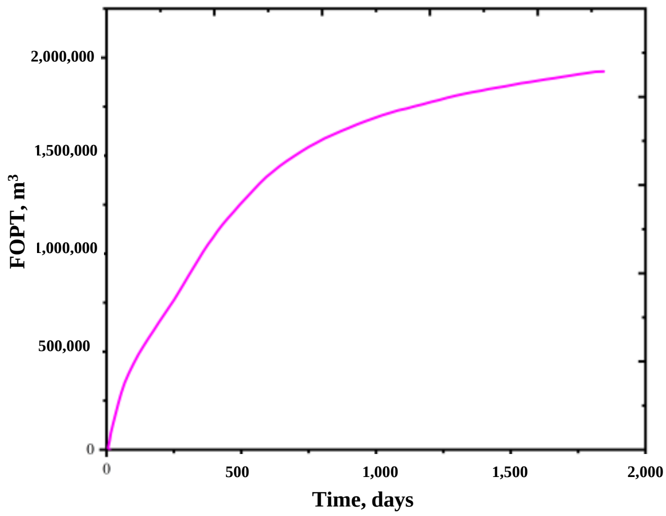

Figure 8. Field oil production total for horizontal wells.

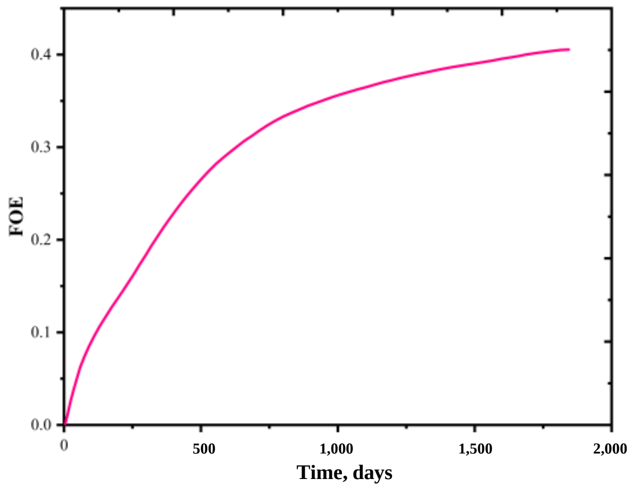

Figure 9. Field oil efficiency for horizontal wells.

3.1.2. Field Water Production

The simulated field water production rate versus time is shown in Figure 10. It can be seen that water production increased with time. Water started to be produced after 180 days. Water cut is the ratio of water produced to the total fluid produced. According to conservation of mass (i.e., material balance) in oil reservoirs, the extracted volume of fluid is relatively constant throughout the well lifetime, but the water share (or water cut) increases with time. As oil is extracted from the reservoir, an increased water cut causes a decline in the oil production flow despite high reservoir pressure. As seen in Figure 11, the water cut increased, which indicates that the injected water broke through and then started to be produced with the oil. After 180 days, water started to be produced with the oil. After water breakthrough, there was a rapid water cut rise, and a slower water cut rise in the late period. The water cut rising curve was convex after water breakthrough.

3.1.3. Field Liquid Production Rate (FLPR)

As production started, the field liquid production rate was constant at 6000 m^3/day up to 30 days, after which it started to decrease slightly, down to 2060 m^3/day at 160 days. It then increased steadily up to 1840 days. From the start of production until the end, the field liquid production rate increased linearly to almost 5,902,705 m^3 at 1840 days, as shown in Figures 12 and 13.

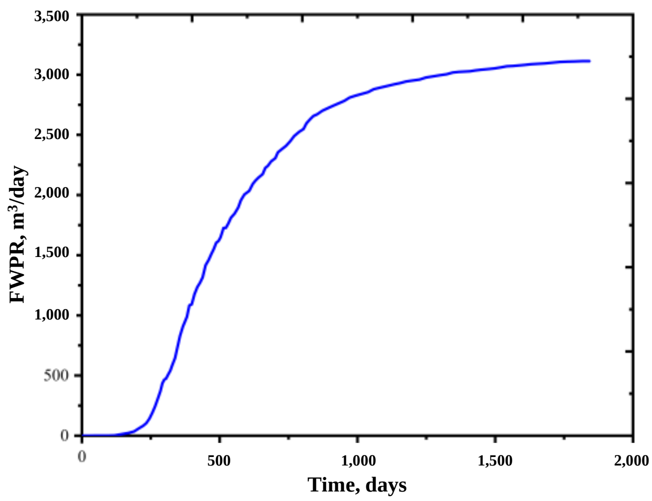

Figure 10. Field water production rate for horizontal wells.

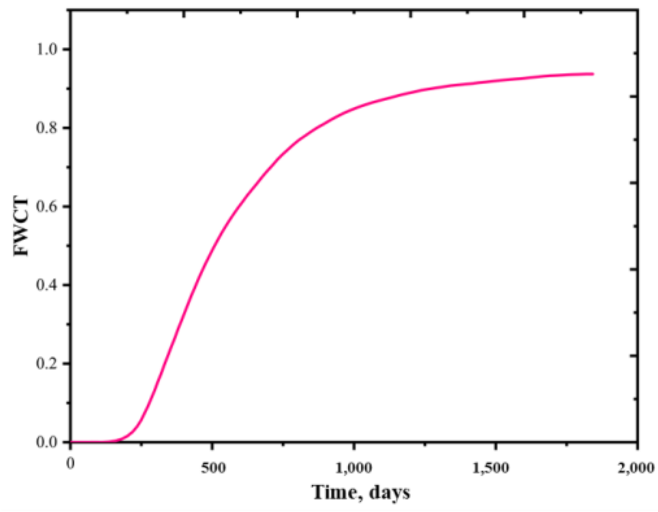

Figure 11. Field water cut for horizontal wells.

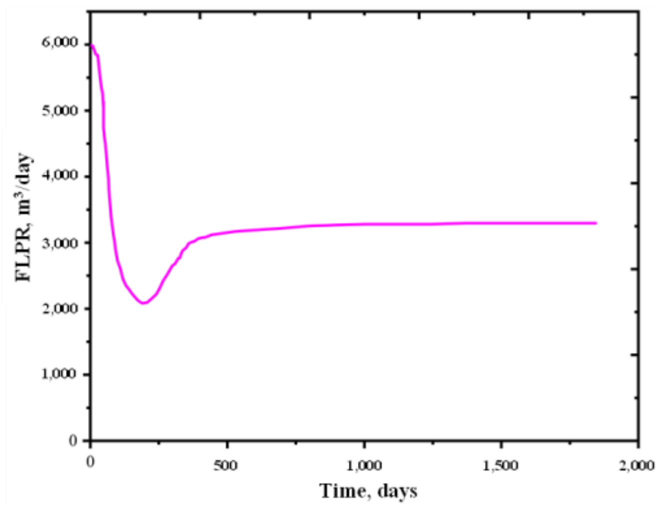

Figure 12. Field liquid production rate for horizontal well.

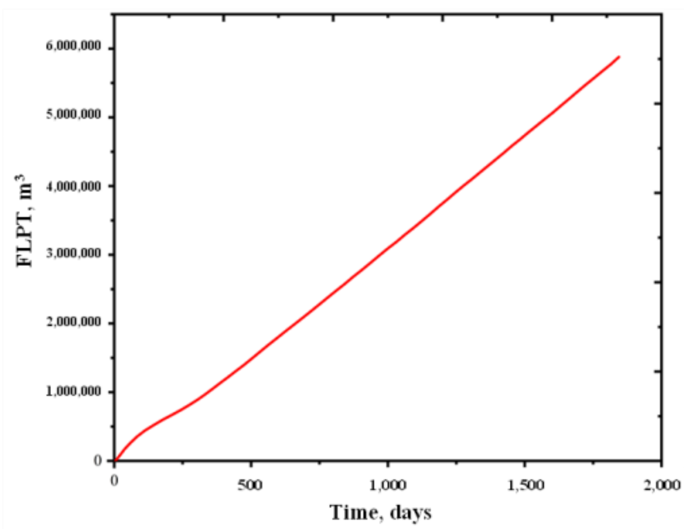

Figure 13. Field liquid production total for horizontal well.

3.1.4. Average Pressure for Field (FPR)

Reservoir pressure dropped rapidly from the initial reservoir pressure of 188 bar to 148 bar and started rising at the start of water injection. Field pressure rose very slowly from 148 bar to a final value of 152 bar in 1840 days. This is normal for a field. A decline in reservoir pressure is due to oil withdrawn from the reservoir being replaced by water encroaching into the oil zone, as shown in Figure 14.

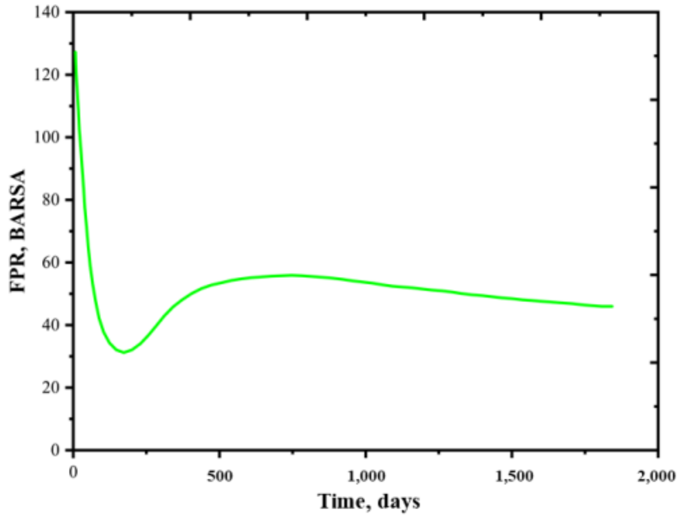

Figure 14. Average pressure for field with horizontal production wells.

3.1.5. Relationship between FOPR and FWPR for Horizontal Wells

As shown in Figure 15, more oil was produced in the early life of the water injection, and this was the primary economic advantage. There was considerable water production with the oil after 180 days. The amount of water increased with the decreasing oil production rate, as water replaced the oil that was withdrawn.

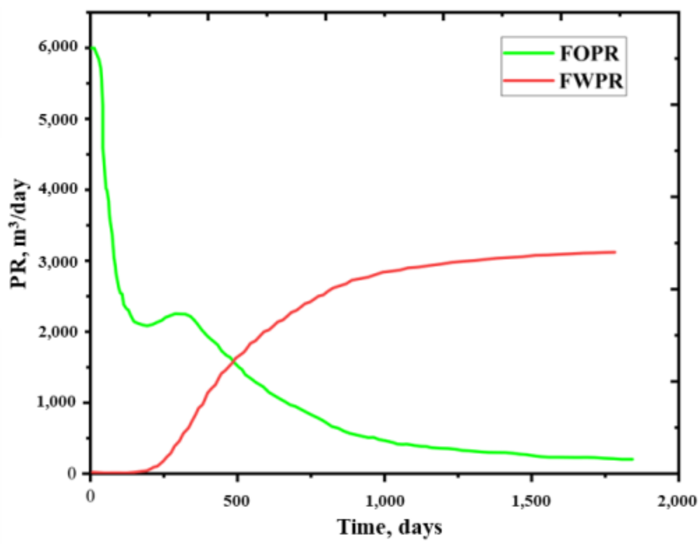

Figure 15. Field oil production rate and field water production against time for horizontal wells.

3.1.6. Pressure Distribution

Figure 16 shows the results before and after 1840 days (5 years) of production. After 5 years of oil production, the simulation was stopped. Figure 16b depicts the final pressure distribution of the simulator. Almost all the pressure in the field was depleted after 1840 days of simulation.

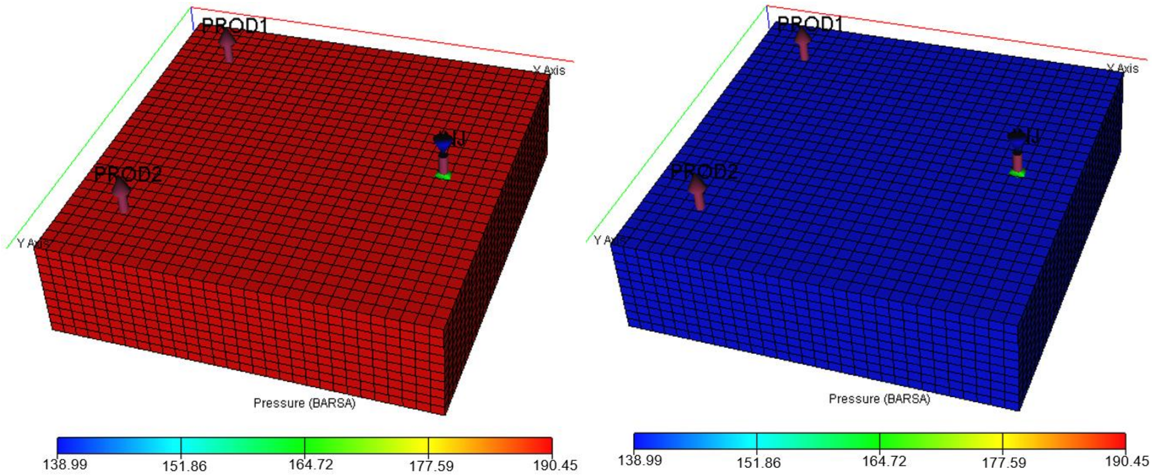

Figure 16. 3D field pressure (**a**) before simulation and (**b**) after simulation of 1840 days.

3.1.7. Saturation Distribution

Even though saturation is not as important as porosity and permeability, the saturation distribution model helps to identify potential high-water areas. Saturation is the fraction of oil, water, and gas found in a given pore space. Figure 17 shows a 3D perspective view of the simulated oil saturation.

As shown in Figure 17b, the injected water has swept the oil into almost the entire reservoir. However, due to permeability variations, only a portion of the oil has been swept into the reservoir. Small portions of the reservoir were left with oil (red color). Some sections of the oil have been replaced by the water near the injection well.

Figure 18 shows a 3D perspective view of the simulated water saturation. The model below reveals that the water saturation distribution is highest (light to dark blue) after water injection for 1840 days. There is high water saturation near the injection well.

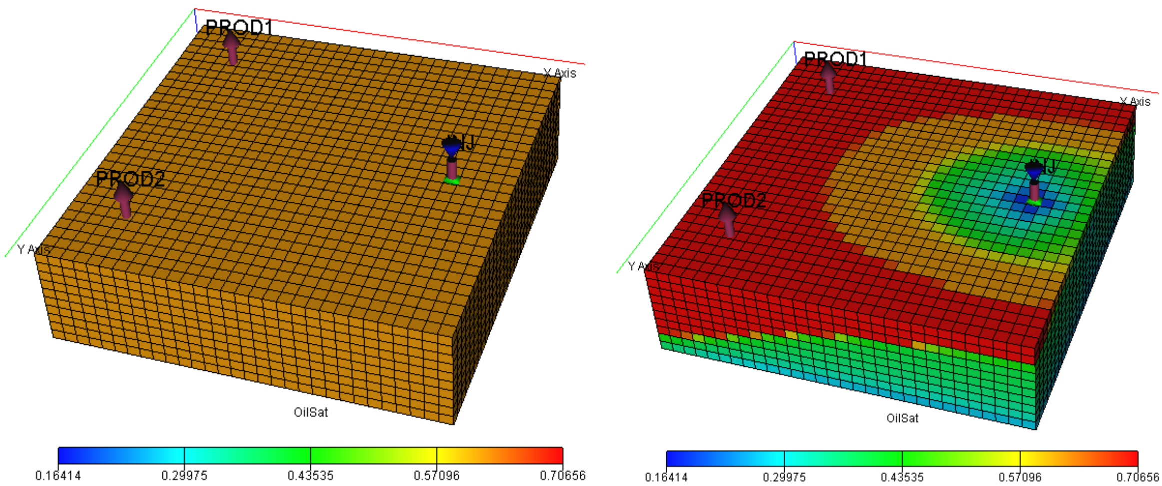

Figure 17. Before (**a**) and after (**b**) simulated 3D oil saturation.

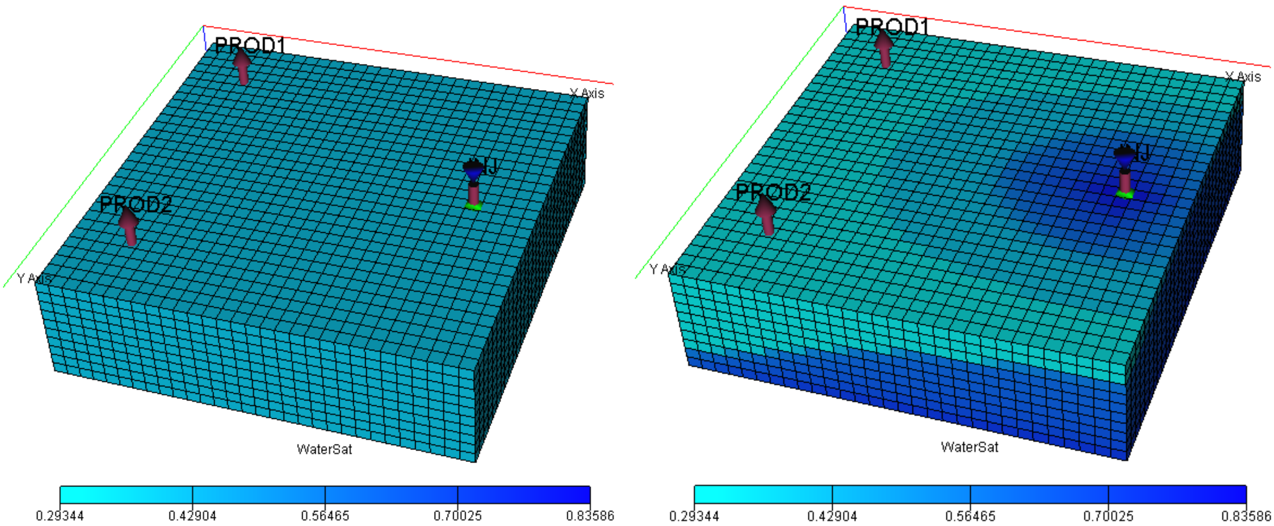

Figure 18. Before **(a)** and after (**b**) 3D simulated water saturation.

3.2. Oil Production from Vertical Wells

3.2.1. Field Oil Production

Figure 19 shows the variation in field oil production rate with production time for vertical wells. The figure shows that the oil production rate decreased from 6000 m^3/day to 186 m^3/day. Figure 20 shows that the cumulative oil production increased as time progressed. It specifies that, at the end of 5 years of production (1840 days), the field oil production total was 1,937,523.9 m^3. Figure 21 shows that after drilling two vertical production wells and one vertical injector well, oil recovery was 39% for 5 years. This indicates that sweep efficiency was poor and a considerable amount of oil was unrecovered.

3.2.2. Field Liquid Production

In Figure 22, we can see that as production started, the field liquid rate production rate was constant at a rate of 6000 m^3/day up to 30 days, and then started to decrease slightly to 2074 m^3/day at 184 days. It then increased slowly up to 1840 days. From the start of production to the end, the field liquid production rate increased linearly to almost 5,939,858 m^3 at 1840 days, as shown in Figure 23.

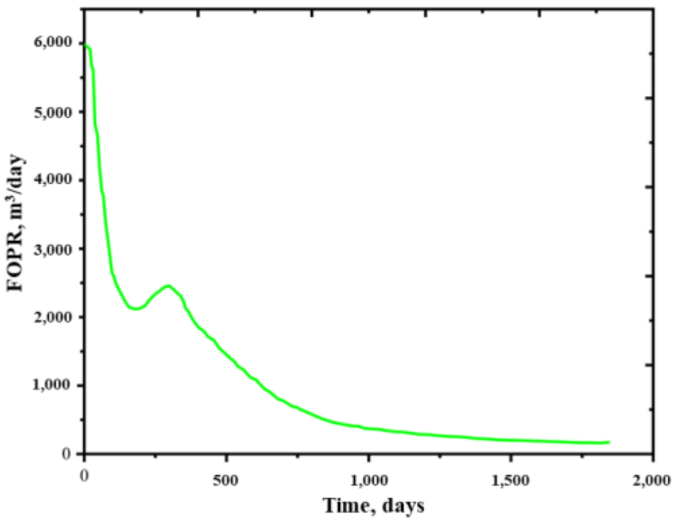

Figure 19. Field oil production rate for vertical well.

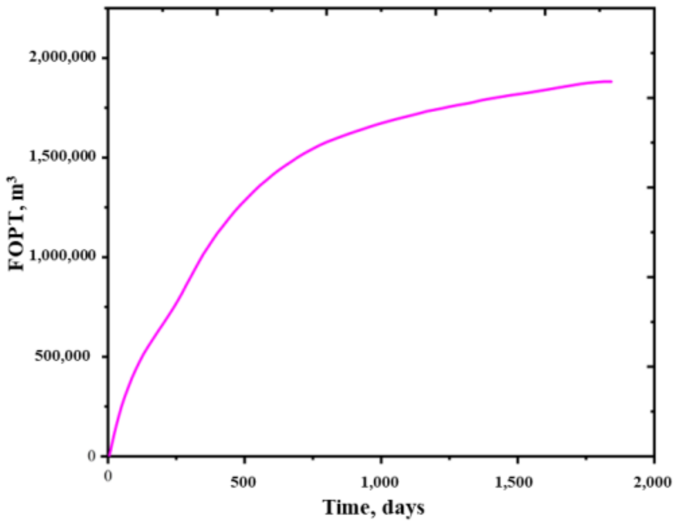

Figure 20. Field oil production total for vertical well.

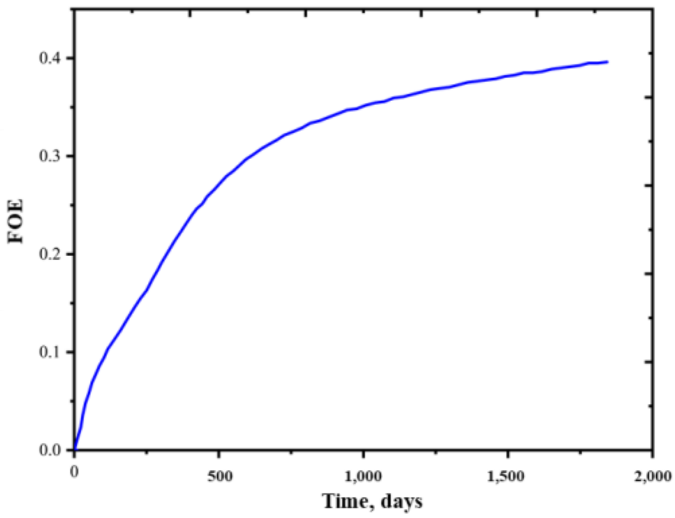

Figure 21. Field oil recovery efficiency for vertical wells.

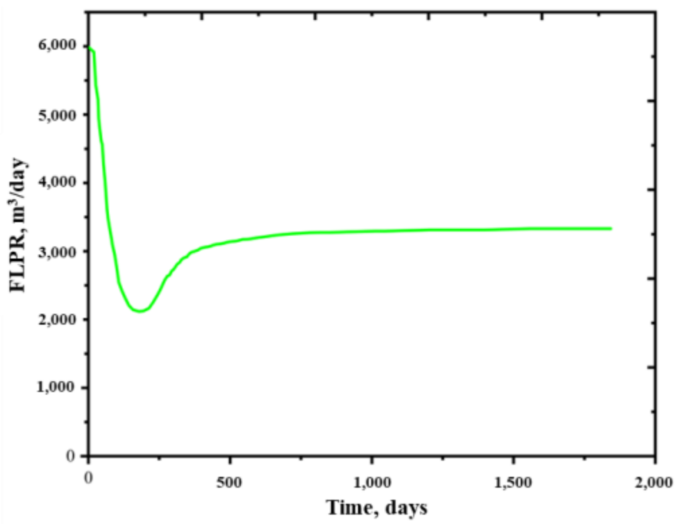

Figure 22. Field liquid production rate for vertical wells.

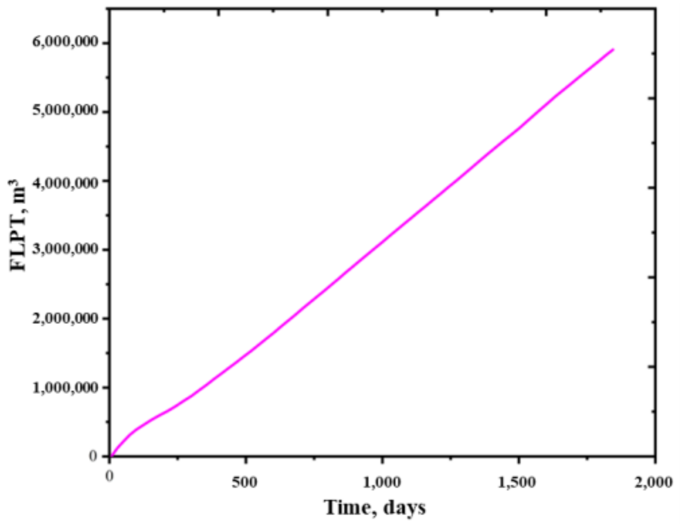

Figure 23. Field liquid production total for vertical wells.

3.2.3. Field Water Cut

Figure 24 shows that there was no water cut between (0–250 days). This means that only oil was produced. After 250 days, the water cut started to increase, which indicated that the injected water had broken through and was being produced with the oil. Figure 25 shows that the water production increased with time. After 180 days, water started to be produced with the oil.

Figure 26 shows that the quantity of water climbed, and often exceeded the volume of the hydrocarbons after 460 days. As reservoirs mature, especially if secondary recovery methods are used, the quantity of water climbs and often exceeds the volume of the hydrocarbons before the reservoir is exhausted.

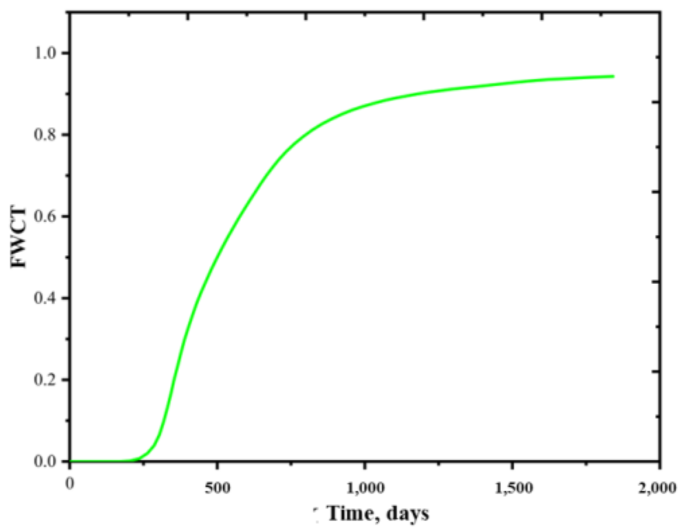

Figure 24. Field water cut for vertical wells.

3.2.4. Saturation Distribution

Figure 27b shows that there was oil that remained in the reservoir after water injection. This remaining residual oil may be recovered by changing the production scheme, such as drilling more injection wells or using improved techniques such as EOR.

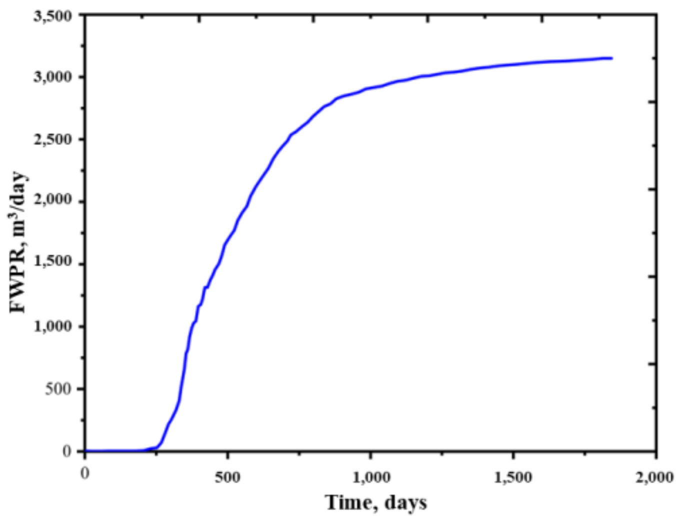

Figure 25. Field water production rate for vertical wells.

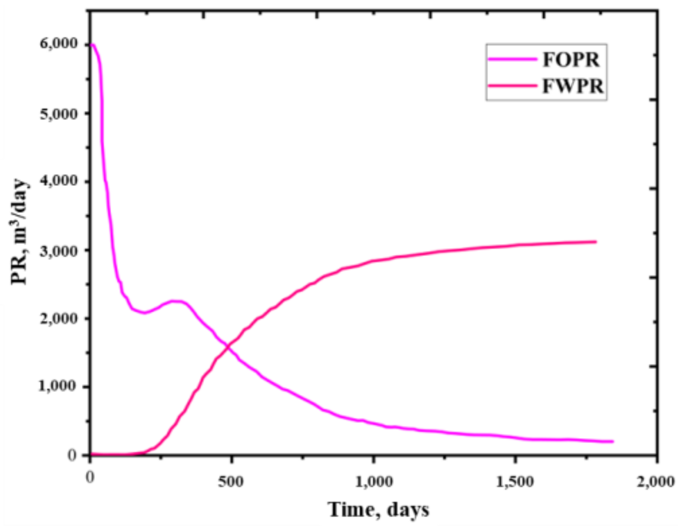

Figure 26. Field oil production rate and field water production rate against time.

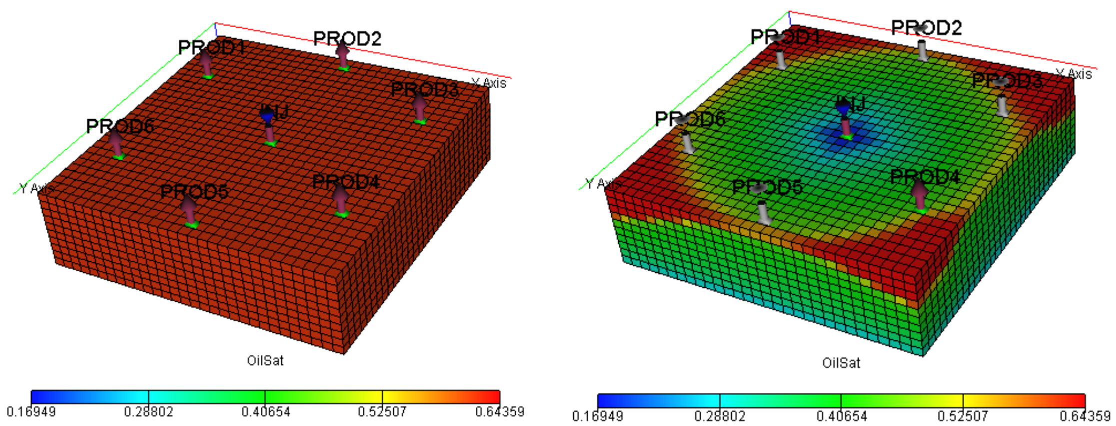

Figure 27. 3D simulated oil saturation (**a**) before and (**b**) after 1840 days of simulation.

Figure 28b depicts an increase in water saturation in production wells as a result of injected water sweeping and replacing the oil.

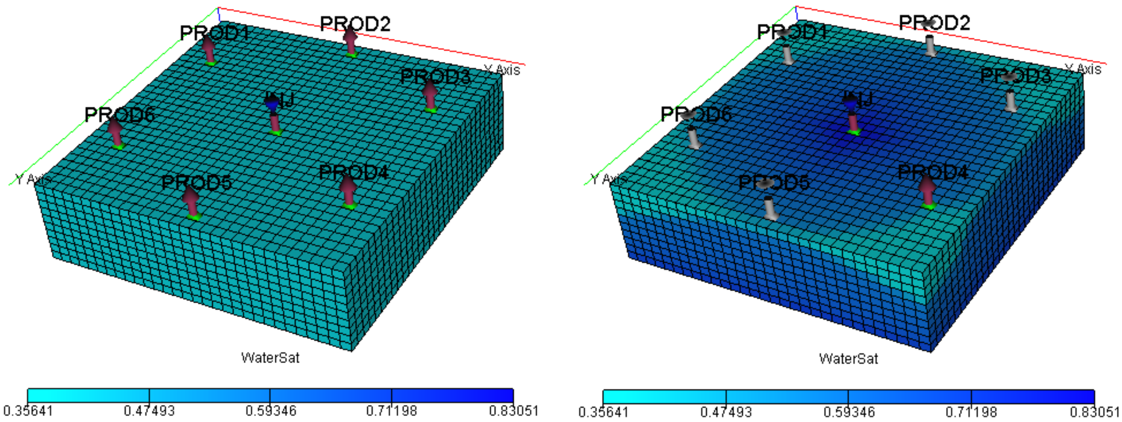

Figure 28. 3D simulated water saturation (**a**) before and (**b**) after 1840 days of simulation.

3.3. Comparison of Vertical and Horizontal Wells

3.3.1. Field Oil Production Total

Figure 29 shows that the field oil production total when drilling horizontal wells was 1,930,000 m^3 after 1840 days, higher than for vertical wells at 1,890,000 m^3. A prominent feature of horizontal wells is the long length of the borehole through the reservoir, which greatly increases the contact area between the well and the reservoir, resulting in higher production per well, faster production, and reduced production time. If the production zone is driven by water, if the oil viscosity is much higher than that of water, a vertical well may encounter the water cone problem. Horizontal wells can deviate in the middle and upper part of the reservoir and then drill a certain length of horizontal section in the production zone. This can not only reduce the possibility of a water cone and prolong the water-free production period, but also the pressure drop per unit length of the producing section is lower than that of a vertical well, as shown in Figure 30.

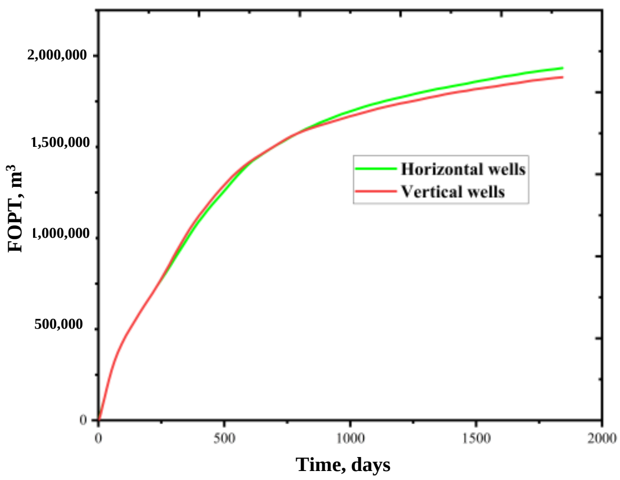

Figure 29. Field oil production total for horizontal and vertical wells.

3.3.2. Total Water Production Rate

Figure 31 shows that the field water production total for vertical wells was 4,100,000 m^3 after 1840 days of production, while for horizontal wells it was 3,900,000 m^3. Vertical wells produced more water than horizontal wells. Horizontal wells minimize water coning effects, hence minimizing water production.

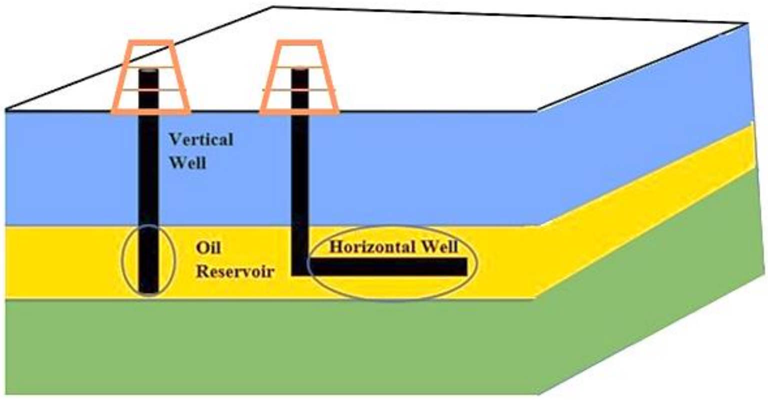

Figure 30. Horizontal well reaching wider section of formation.

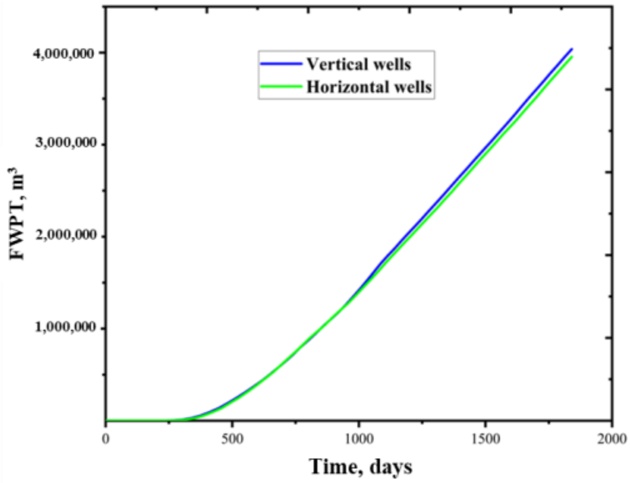

Figure 31. Comparison of field total water production rate for vertical and horizontal wells.

3.3.3. Oil Recovery Efficiency

Figure 32 indicates an oil recovery of 41% for the field with horizontal wells compared to 39% for vertical wells. Both wells have a less than 50% recovery factor. This means that a considerable amount of oil was unrecovered or trapped in the reservoir.

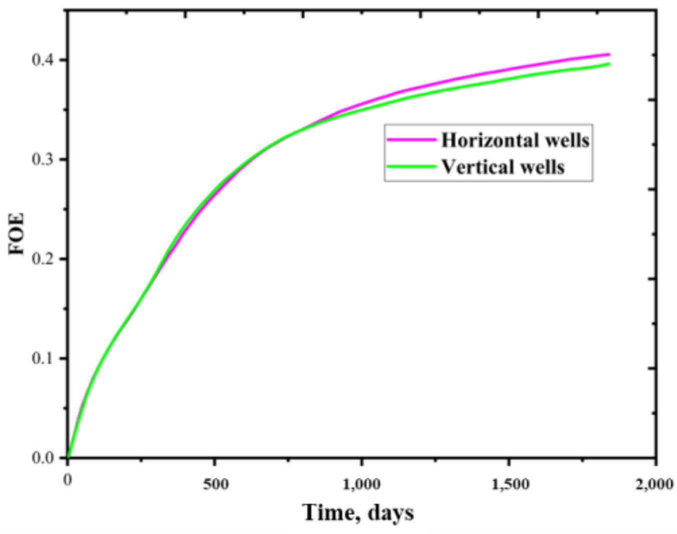

Figure 32. Field oil efficiency of vertical and horizontal wells against time.

4. Discussion

(1) In order to increase horizontal well productivity and raise field oil recovery above 50%, the horizontal well length should be increased to a larger area of the reservoir section. The vertical well productivity index may not be affected by well length. The oil production rate for horizontal wells was very high compared to that of vertical wells, in such a way that the horizontal wells were able to produce more oil for another year (if the production was extended) while the vertical wells could not.

(2) The grid steps were somewhat large in the conceptual simulation models in this study, which could be used for a mechanism study on the performance comparison between horizontal wells and vertical wells; to acquire more exact results and make the simulation more efficient, a proper grid step is necessary.

5. Conclusions

According to the simulation of the conceptual model and the discussion explained in this research, the following conclusions may be drawn:

(1) After 1840 days, horizontal wells produced 1,930,000 m^3 of oil, while vertical wells produced 1,890,000 m^3 of oil. The total oil produced in the field by horizontal wells was 40,000 m^3 more than that produced by vertical wells for 1840 days. This showed that horizontal wells had better performance than vertical wells in terms of productivity. Field oil recovery for horizontal and vertical wells was 41% and 39%, respectively. This indicated that horizontal wells have higher oil recovery rates than vertical wells, but both wells' configurations had less than 50% recovery efficiency, which meant that a considerable amount of oil was unrecovered in the reservoir.

(2) The field water cut for horizontal wells was lower than that for vertical wells. Therefore, it is suggested to use horizontal wells for oil production activities in order to obtain a high oil recovery in a short period of time. In spite of the fact that horizontal wells had a much greater extent of oil recovery than vertical wells, their ratio still was not in favor with respect to the remaining oil. This was due to the minimum rate of water injected into the field. Therefore, injection wells should be added so as to recover a reasonable amount. To increase the effect on oil production, the water injection rate should be increased to a reasonable rate in order to achieve an extensive effect on pressure and sweeping efficiency.

(3) The number of injection wells should be increased and well patterns and locations should be redesigned. Due to the low oil recovery in the simulation, to evaluate the potential for increasing oil production, an EOR screening should be performed and surfactant or polymer flooding should be chosen as good candidates.

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Nomenclature

S_w	Water saturation	R_s	Ratio of solution—gas and oil
S_g	Gas saturation	B_o	Oil volume factor
K_{rw}	Relative permeability of water phase	B_g	Gas volume factor

K_{rg}	Relative permeability of gas phase	μ_o	Oil viscosity
K_{ro}	Relative permeability of oil phase	μ_g	Gas viscosity
P_{bub}	Bubble point pressure	P_c	Capillary pressure

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